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ESSENCE AND POTENTIALITY: ARISTOTELIAN STRATEGIES OF ADDRESSING PROBLEMS OF CHANGE AND PERSISTENCE

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by

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Dedication

For my long-suffering wife Jennifer.

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When Aristotle makes his case that time is a property of motion, he not only argues that time depends for its existence on motion, but that it derives its structural properties from motion as well. But if this is to avoid a vicious circularity, then motion cannot presuppose time, and the order of motion must be definable in abstraction from the order of time. I argue that Aristotle is able to do exactly this, based upon his theory of act and potency (*energeia* and *dunamis*), and upon the theory that all natural change is teleological. I propose that a linear order may be defined on the phases of a change, using the relation "*x* is potentially *y*," where *x* and *y* range over different phases of an Aristotelian natural substance (e.g., Socrates-as-a-boy, Socrates-as-a-man, etc.). This is possible, I claim, because a special asymmetric potentiality is involved which marks out the

stages of a change as prior and posterior based upon their proximity to a given goal, rather than upon their order in a temporal sequence. I also argue that if *x* and *y* appear in states of affairs that obtain at different times, then the "*x* is potentially *y*" relation provides a criterion for diachronic identity, since it relates a single entity at one time to *itself* at another time. Moreover, I argue, based on an account that takes forms to be individuals that persist over time, that the forms which give substances these special potentialities are early analogues of the individual essences proposed by the Stoics and by Duns Scotus as criteria for identity, and by contemporary metaphysicians such as Kaplan and Plantinga to secure identity across possible worlds. I look at two ancient puzzles about persistence, viz., the Growing Argument by Epicharmus, and a similar puzzle about alteration mentioned by Aristotle in *Phys*. 4.11, and assess the adequacy of Aristotle's criterion of identity for solving them. As a point of comparison, I also assess the solution to the Growing Argument proposed by the Stoic philosopher Chrysippus, which features a *reductio ad absurdum* of certain premises of these puzzles.

Table of Contents

Introduction

In the first three books of the *Physics*, Aristotle sets forth a metaphysic of change that has three important features. First, he claims that change consists in the passage between contrary termini. Second, he claims that the *terminus ad quem* of a change is a goal, which either constitutes, or contributes to the perfection of the nature of the changing thing. And third, he claims that the termini of a change are related by the concept of potentiality, where the changing thing in the *terminus a quo* is potentially the changing thing in the *terminus ad quem.*

In the sequel, I will consider a few of the virtues and shortcomings of this metaphysical scheme. In particular, I will show how the teleological aspect of it can be used to account for the order and direction of time, and how the concept of potentiality may be used to supply a criterion of identity. I will also show how the idea that change consists in the passage between contrary termini makes it difficult for Aristotle to account for how motions come to be and pass away, and to conceive of continuous changes in velocity.

In Chapter 1, I prove that Aristotle has the resources within his philosophy to answer a common criticism of his theory of the order and the direction of time. In the 4th book of the *Physics*, Aristotle appears to derive the order of time from the order of change in something like a one-one, structure-preserving mapping from the phases of a change to the instants in which those phases occur. Aristotle thinks that he can do this, because he claims that time is a property of motion, and

is ontologically dependent upon motion. I refute the claim that this derivation is viciously circular. Now, if we took the modern "at-at" view of motion, in which motion consists merely in being in different places at different times, then clearly, the derivation *is* circular. And although most scholars would agree that Aristotle thinks that motion is more than being at different places at different times, there is, nonetheless, a widely held belief that, no matter what Aristotle thinks that change is, he cannot, in principle, separate time from change even in thought.

Now, I think that the "at-at" view is an excessively abstract way to think about motion, especially if we want to be fair to Aristotle, which is to say, if we want to allow Aristotle the full resources of his own philosophy to answer such an objection. The resource that I have in mind, in particular, is Aristotle's view that moving things have natures. Now, for Aristotle, a nature is something that governs the characteristics of a motion of a thing that possesses it. In Aristotle's universe, moving things behave the way they do, not because they have certain common properties such as mass, or need to obey certain universal mechanical laws, but because they have individual, substance-specific natures, which entail certain characteristic potentialities, developmental and behavioral potentialities, as well as an impulse or a drive to fulfill those potentialities. For instance, fire has a special teleological potentiality to be at the periphery of the universe, as well as a drive that will move it there, if nothing hinders it. Similarly, earth has a special teleological potentiality to be at the center of the universe, and it has an inner drive that will move it there, if nothing hinders it. And the same can be said for more complex organisms. Take a chicken, for instance. Why does a chicken

cross the road? Well, Aristotle would say that it is probably because it sees a pile of chicken feed on the other side, and has formed the goal to go over and eat it. Why do chickens eat chicken feed? Because their nature is such as to require a certain sort of nourishment. And why is that? Because this sort of nourishment is required for their biological development, or the perfection of their substantial form. In other words, Aristotle claims that chickens are born, as biological natural substances, with a nature, which entails certain developmental potentialities, and if those developmental potentialities are to be realized, then a certain nourishment is required, and if a certain nourishment is required, then certain behaviors are required to procure it. So that is why the chicken crosses the road. Now, what this means is that, in Aristotle's world, things, in general, have teleologies. They have things that they want to do, and things that they *will* do if you do not hinder them. The way that this helps Aristotle is that he can order the phases of a change with reference to a given goal instead of with reference to a temporal series. The phases of an Aristotelian change, in other words, have a normative order, and this normative order can be relied on when deriving the order of time from the order of change. And since the normative order has no explicit time references, then Aristotle's derivation is not circular.

In Chapter 2, I deal with another philosophical problem that is suggested by a device that I use to think about the order of time. The modern way to think about orders is in terms of domains and relations, and in order to make what I have been outlining more perspicuous, I define a relation on Aristotle's behalf. The relation is x is potentially y, where x and y range over the phases of an

Aristotelian natural substance. Phases might be something like Socrates-as-a-boy or Socrates-as-a-man. If *x* is Socrates-as-a-boy and *y* is Socrates-as-a-man, then the relation holds because Socrates as a boy has the potentiality to develop into Socrates-as-a-man. Now a question arises because, if *x* and *y* are phases of a change of an Aristotelian substance, and *x* and *y* exist at different times, then the holding of the relation "*x* is potentially *y*" also seems to be a sufficient condition for diachronic identity or identity over time. The reason for this is that the relation *x* is potentially *y* relates a substance to *itself* at another phase. It relates Socrates-as-a-boy to Socrates-as-a-man, for instance. And this makes sense because we would probably think that Socrates-as-a-boy has the potentiality to become Socrates-as-a-man, but not Plato-as-a-man. And why is that? It is because Socrates' potentiality is constrained somehow to prevent this. There are certain things that Socrates-as-a-boy has the potentiality to become and Plato-as-a-man is not one of them. This seems reasonable, but why it should be so is not entirely clear. What is needed is a proper criterion for diachronic identity; one that appears to justify the identification of Socrates-as-a-boy and Socrates-as-a-man.

The criterion that I offer on Aristotle's behalf is a variation on one suggested by Michael Frede. Frede suggests that Aristotle should use the continuity of spatio-temporal histories as a criterion of diachronic identity. I suggest, rather, that it is the accumulation of time-indexed properties that serves this function. I claim that to have a spatio-temporal history is just to have a collection of time-indexed properties, and to have a unique spatio-temporal

history is just to have at least one property contained in that history that is not shared with any other individual. What is attractive about this proposal is that at one stroke it provides a criterion for both individuation and identity over time. If Socrates and Callias have type-identical species forms that are instantiated at different places at time *t*, this fact not only differentiates them at that time, but because they continue to possess the property at future times of having been in distinct places at time *t*, then they are also differentiated at all future points in time. Moreover, the fact that they possess this property at all future points in time, allows them to be reidentified as the individuals with these particular spatio-temporal histories. This makes spatio-temporal histories individual essences, not in the Aristotelian sense of something that expresses what *sort* of thing one is dealing with (for instance, a man or a horse, etc.), but an essence in the sense of a property that a thing has in every circumstance in which it exists.

Finally, in Chapter 3, I show how the idea that motion is simply the traversal from terminus A at time *t* to terminus B at time *t* + 1, combined with Aristotle's strict mathematical finitism, makes it impossible for Aristotle to account for how motions come to be and pass away. In particular, I argue that since Aristotle rejects instantaneous velocities, he is faced with a dilemma in accounting for the way in which a moving body comes to rest, for instance, at the end of a period of motion. Either he must say that the moving body must traverse the whole of an infinite sequence of non-instantaneous velocities in the course of coming to rest, or he must say that it must traverse a finite sequence of non-instantaneous velocities and then transition to the period of zero velocity

discontinuously. Accepting either horn of the dilemma would be devastating for Aristotle. If he accepts the traversal of an infinite sequence he must abandon the view that finite human minds are up to the task of understanding the universe, which rests on the assumption that the universe, in its essence, is a finite place containing finite things. If he accepts a discontinuity in the passage between a period of motion and a period of rest, however, I will argue that he must accept a similar discontinuity in the passage between *any* two velocities, and this will result in motion being pervasively discontinuous, not just at its beginning and end.

Chapter 1: The Order and Direction of Time

1.0 The Order of Change

This chapter is about an aspect of time that has interested philosophers since at least Aristotle, and that still generates a large amount of scholarship. It is the problem of the order and direction of time, and it arises because certain processes seem only to proceed in one direction. For instance, when one pours milk into a cup of coffee, it always spreads out rather than concentrating in one spot. And if one heats up a pan and immerses it in cold water, the water always becomes warmer, and the pan always becomes colder. It is never the other way around. Philosophers have long suspected that apparently lawlike asymmetries in changes like these imply an asymmetry in time itself, and some have even argued that this asymmetry in time *derives* from an asymmetry in changes. In other words, the claim is that one time is before or after another *because* one event or state is before or after another. Aristotle takes precisely this view in the 4th book of the *Physics*, where he says that "The distinction of 'before' and 'after' holds primarily, then, in place; and there in virtue of relative position. Since then 'before' and 'after' hold in magnitude, they must hold also, in an analogous fashion, in movement. But also in time the distinction of 'before' and 'after' must hold, for time always follows movement." The claim that time follows movement, here, amounts to the claim that one time is before or after another *because* one phase of a change is before or after another.

I will not go into Aristotle's justification for this, except to say that it is based upon the view that time is a property of change, which makes it dependent upon change in all of the ways that a property is dependent upon its subject. What I am going to address here is the common criticism that Aristotle's view contains a vicious circularity. Now, if we take the view that motion consists merely in being in different places at different times, then the derivation obviously *is* circular, because, in that case, the order of change is defined in terms of the order of time. Gwil Owen argues that Aristotle has no choice but to define the order of change in this way, and tries to prove this by envisaging motion along a linear magnitude between points A and B, for instance.[1] He claims that one can define two orders on the magnitude, either from A to B, or from B to A, but one cannot select one of the two orders as the order of the change without importing a time reference. In other words, to say that the motion begins at point A is just to say that the moving body is at position A prior to the time at which it is at position B.

1.1 The normative order of change

There is a widespread consensus today, and perhaps even an orthodoxy, that this argument, or others like it, succeed in convicting Aristotle of a vicious circularity. One sees this view, for instance, in articles by Julia Annas, Sarah Broadie, Denis Corish, and Richard Sorabji, and I am not aware that anyone has tried to challenge it.[2] As a matter of fact, however, Owen's objection only *appears* to succeed because he has already assumed a very abstract picture of motion. And I think

that if we look closely at some of Aristotle's more concrete discussions of how changes get their orders, we will see that he has the resources to specify an order of change without a reference to times.

This seems to be the case, for instance, in three passages dealing with the biological development of a boy into a man. The first is at *Metaphysics* Delta 11, 1018b19-21, where Aristotle explains what it means to be prior in a change: "Other things are prior in change; for the things that are nearer the first mover are prior (e.g., the boy is prior to the man); and the prime mover is also a beginning absolutely." He distinguishes, here, two senses in which the term "prior" might be taken; one in the sense in which the prime mover is prior, that is, as the ultimate final cause of all change, and another in the sense that the boy is prior to the man. The second passage is at *Generation of Animals* 724a22-3, where we see that the boy is prior to the man in the sense of an order of succession, and that we can also express this priority by saying that the man comes *from* the boy, using the Greek preposition ἐκ:

> "Now we speak of one thing coming from (ἐκ) another in many senses; it is one thing when we say that night comes from day or a man becomes man from a boy, meaning that the one succeeds the other (τόδε μετὰ τόδε); it is another if we say that a statue is made from bronze and a bed from wood."

But if we look at a third passage, at *Metaphysics* 994a22-994b3, it is clear that this order is not a *temporal* succession:

> One thing comes from another in two ways—not in the sense in which "from" means "after" (as we say "from the Isthmian games come the Olympian"), but either (i) as the man comes from the boy, by the boy's changing, or (ii) as air comes from water. By "as the man comes from the boy" we mean "as that which has come to be from that which is coming to

> be" or "as that which is finished from that which is being achieved" (for as becoming is between being and not being, so that which is becoming is always between that which is and that which is not; for the learner is a man of science in the making, and this is what is meant when we say that from a learner a man of science is being made); on the other hand, coming from another thing as water comes from air implies the destruction of the other thing. This is why changes of the former kind are not reversible, and the boy does not come from the man (for it is not that which comes to be something that comes to be as a result of coming to be, but that which exists after the coming to be; for it is thus that the day, too, comes from the morning—in the sense that it comes after the morning; which is the reason why the morning cannot come from the day); but changes of the other kind are reversible.

The boy precedes the man in time, but that is not the point, as is clear from the phrase "not in the sense in which 'from' means 'after' (as we say 'from the Isthmian games come the Olympian')." The point, rather, is a metaphysical one about the relationship between boyhood and manhood. Aristotle says "by 'as the man comes from the boy' we mean 'as that which has come to be from that which is coming to be' or 'as that which is finished from that which is being attained (for as becoming is between being and not being, so that which is becoming is always between that which is and that which is not.'" Evidently, the point of identifying "that which has come to be" and "that which is finished" with the man, and "that which is coming to be" and "that which is being attained" with the boy, is to mark the man off as something complete and to mark the boy off as something in process and not yet complete. If the man is complete, then manhood must be the goal or culmination of the boy's development, and if incompleteness is assessed negatively, then manhood will be a *normative* goal. Indeed, identifying manhood with a state of being and boyhood with a state between being and non-being seems to confirm this negative assessment by making boyhood a state of

privation. Manhood is a better or more perfect state than boyhood, and that is why it has the status of a goal.

Now clearly, if we can order the phases of human biological development in terms of "good," "better," and "best" we need not also give them a temporal ordering. It may be the case that the good state is always earlier, and the best state is always later, but the charge of circularity rests on our alleged inability to conceive of this ordering in anything but temporal terms. If we can conceive of this order abstractly, attending only to the normative relationships between the phases, then I think that Aristotle will have escaped the charge of circularity.

There are similarities between what I am suggesting and Leibniz's causal theory of the temporal order. Leibniz sought to reduce time to the order of non-contemporaneous events, and he sought to derive this order from the non-temporal order of cause and effect: "When one of two non-contemporaneous elements contains the ground for the other, the former is regarded as the antecedent, and the latter as the consequent. My earlier state of existence contains the ground for the existence of the later."[3] Leibniz, based upon certain metaphysical assumptions about the nature of individual substances, claims a certain asymmetric and necessary relation between causes and effects. Likewise, Aristotle, based upon his theories of nature, final cause, actuality and potentiality, asserts a certain asymmetric and necessary relation between the phases of a change.

1.2 The relation *x* is potentially$_{\beta}$ *y*

The metaphysical basis for this normative order lies in Aristotle's concepts of nature and potentiality. The important discussions of nature are in *Physics*, book 2, chapter 1 and *Metaphysics* book Delta, chapter 4. In the latter chapter, Aristotle tells us that a nature, in the primary sense of the word, is a form or essence, and then he identifies the form or essence with a source or a principle of change that is innate in a natural substance. In *De anima*, book 2, Aristotle identifies the human form with a person's soul, and he also claims that the soul is the efficient cause of a person's movements, so one can see how the soul might be the boy's nature in the sense of his form or essence, as well as in the sense of the innate impulse that drives his development. But Aristotle also says, and this is the part that is relevant to a normative ordering, that a nature is a form that is attained or realized through a process of growth or development. Now obviously, if the boy's nature just consists in having a human form or soul, then there is no need for him to attain what he already has. And if we assume that the attainment in question is growing to manhood, then the boy's nature must consist in having a human soul that is distinguished in some way from a man's, and this difference must be a deficiency if becoming a man is to be an attainment. And, indeed, this appears to be the case, since, according to Aristotle, a boy has an underdeveloped rational faculty. Aristotle claims in *Nicomachean Ethics* book 4, chapter 12, that boys "live at the beck and call of appetite" and it is their tutor that assumes the role of their rational faculty, while their own is not yet developed. And we can see from *Politics* 7.15, 1334b15-25 that the soul of a man with a fully functioning

rational faculty is the end toward which the boy's nature is striving, and that this attainment is always preceded by the development of the irrational faculties:

> Now, in humans reason and mind are the end toward which nature strives. … As the body is prior in order of generation to the soul, so the irrational is prior to the rational. The proof is that anger and wishing and desire are implanted in boys from their very birth, but reason and understanding are developed as they grow older. (*Pol.* 7.15, 1334b15-25)

The nature of the boy, then, is exemplified in a human soul that is partially developed relative to the goal state of manhood. A nature, then, sets the normative order in the biological development of the boy by specifying the goal state of possessing a fully functioning rational faculty, and supplying the efficient cause, in the form of a principle of motion, for moving this development along.

Now the way that this notion of incompleteness, or partial development relative to a goal gets cashed out in Aristotle is in terms of special teleological potentialities. A boy has many potentialities. He has the potentiality to win a marathon or to die in battle, for instance, but the only potentiality that makes him a partially developed man is the potentiality to have a fully functioning rational faculty. In the *Generation of Animals* book 2, chapter 3, Aristotle tells us that the unfolding of the process of biological development represents the realization of special teleological potentialities that were present in prior stages of the process. For instance, the nutritive soul exists potentially in the semen (σπέρμα) and in the dormant[4] embryo (κύημα: *GA* 32.3 736b8-10). The sensitive/appetitive soul exists potentially in the active[5] embryo (κύημα) but actually in the infant and child. And the rational soul exists potentially in the boy, but actually in the man, which is the end toward which the whole process is directed.

Further light is shed on these teleological potentialities in the following passage from the *Metaphysics*:

> But we must distinguish when a thing exists potentially and when it does not; for it is not at any and every time. E.g. is earth potentially a man? No—but rather when it has already become sperm, and perhaps not even then. … In the cases in which the source of the becoming is in the very thing which comes to be, a thing is potentially all those things which it will be of itself if nothing external hinders it. E.g. the seed is not yet potentially a man; for it must be deposited in something other than itself and undergo a change. But when through its own motive principle it has already got such and such attributes, in this state it is already potentially a man; while in the former state it needs another motive principle, just as earth is not yet potentially a statue (for it must first change in order to become brass.) (*Metaph.* Book Θ, chapter 7 1048b37-1049a18)

Now the first thing that Aristotle does, in this passage, is to deny that earth or sperm is potentially a man, because, as he explains a little further on, first "[sperm] must be deposited in something other than itself and undergo a change." As I just noted, the nutritive soul exists potentially in the semen (σπέρμα), but it appears that this is not enough to make the semen potentially a man. "But," Aristotle continues, "when through its own motive principle [the semen] has already got such and such attributes, in this state it is already potentially a man; while in the former state it needs another motive principle." The idea, here, I think, is that when the sperm acquires "such and such attributes" and "its own motive principle," it acquires a nature in the sense of an actualized form and an innate principle of motion. Although an active embryo has only a nutritive soul in actuality, and, as Aristotle says in the *Generation of Animals* book 2, chapter 3, it lives the life of a plant, it has a nature in the form of a partially developed human form or soul. This soul, due to its nature and partial development, has certain

characteristic developmental potentialities, and because it has an innate impulse of change that is directed at realizing these potentialities, then it is "potentially all those things which it will be of itself if nothing external hinders it."

Now since the nutritive soul exists potentially in the semen (σπέρμα) and in the dormant embryo (κύημα) prior to the stage in which it acquires a nature and begins to develop, it seems that we can make a distinction, here, between two types of potentiality. On the one hand, the potentiality of the semen would appear to be a fitness to be endowed with a nature.[6] On the other hand, the potentiality of the developing embryo to acquire a sensitive and appetitive faculty and the child to acquire a rational one, would appear to be the potentiality, once endowed with a nature, to perfect it. Let us call the fitness to be endowed with a nature potentiality$_{\alpha}$ and the potentiality, once endowed with a nature, to perfect it potentiality$_{\beta}$. Potentiality$_{\beta}$ is the basis for the ordering of the phases in the biological development of a boy into a man. Or to put it more formally, if the relation *x* is potentially$_{\beta}$ *y* holds of two phases of this change, where phases are things like Coriscus-as-a-boy and Coriscus-as-a-man, for instance, then the phases of the change are in their correct normative order. If the relation does not hold, then the phases are not correctly ordered. And going back to the claim in passage one that the order of time is derived from the order of change, we can see that what Aristotle is basically saying is that if the relation *x* is potentially$_{\beta}$ *y* holds of two phases of this change, then the *x* will be the earlier phase and the *y* will be the later one, and that, moreover, the relation *x* is earlier than *y* holds in virtue of *x* is potentially$_{\beta}$ *y* holding.

So where *y* represents a phase of a substance with a perfected substantial form, saying that *x* is potentially$_{\beta}$ *y* means that *x* is a phase of that substance that is incompletely actual relative to *y*, and that, moreover, *x* is moving toward becoming *y* by means of an innate impulse of change that is specifically directed to this purpose. This also means that something that is potentially$_{\beta}$ *y*, has all of the stages in its development — that is, "all those things which it will be of itself if nothing external hinders it" — defined in terms of its eventually being *y*. In other words, Coriscus-as-an-infant is potentially$_{\beta}$ Coriscus-as-a-toddler, as well as potentially$_{\beta}$ Coriscus-as-a-teenager, but since these are all stages on the way to Coriscus-as-a-man, they are potentialities$_{\beta}$ for incomplete actualities (incomplete because they fall short of being Coriscus-as-a-man), and thus are, in a way, incomplete potentialities$_{\beta}$ themselves. Thus, on the assumption that becoming *y* is to be identified with the stages passed through on the way to this goal,[7] Aristotle makes the claim that "change is thought to be a sort of actuality, but incomplete, the reason for this view being that the potentiality whose actuality it is is incomplete" (*Phys*. 201b31-33).

Accordingly, Aristotle's full-dress definition of change, in *Physics* book 3, chapter 1 ("change is the actuality of what exists potentially, *qua* existing potentially"), may be interpreted as follows: Change is an actuality of what exists potentially$_{\beta}$. And an actuality of what exists potentially$_{\beta}$ is an incomplete actuality of *y* or an actuality that is still potentially$_{\beta}$ *y* (that is, an incomplete actuality of an incomplete potentiality$_{\beta}$). Thus, it is an actuality *qua* existing potentially$_{\beta}$ (that is, *qua* still potentially$_{\beta}$ *y* and not yet actually *y*).[8]

1.3 Conditions to be satisfied

In order for the relation *x* is potentially$_\beta$ *y* to order the prior and posterior in change in a strict linear order, it must be transitive, asymmetric, and connected. According to the standard definitions of these properties, a relation is transitive if and only if: if *x* is potentially$_\beta$ *y*, and *y* is potentially$_\beta$ *z*, then *x* is potentially$_\beta$ *z*; a relation is asymmetric if and only if: if *x* is potentially$_\beta$ *y*, then *y* is not potentially$_\beta$ *x*; a relation is connected if and only if, for every *x* and *y*, if $x \neq y$, then either *x* is potentially$_\beta$ *y* or *y* is potentially$_\beta$ *x*. A familiar example of a strict linear order is the domain of the real numbers ordered by the "less than" relation. Transitivity and asymmetry should be familiar, but connectedness, perhaps, less so. Connectedness ensures that no elements in the domain are left out of the order, and that no branching occurs. For instance, suppose we have three distinct real numbers *a*, *b*, and *c*, and that *b* and *c* are related to *a* by the "less than" relation. In this case, the connectedness of the "less than" relation ensures that either *b* is less than *c* or *c* is less than *b*.

Presumably, the relation should also hold necessarily, not in the sense that there must necessarily be some *y* for every *x* such that *x* is potentially$_\beta$ *y* (since a boy may die and fail to become a man, or if *x* is a man, he is not also potentially$_\beta$ a man), but in the sense that if *x* and *y* are distinct phases in the development of a single subject, these phases must necessarily be related by this relation. It cannot be a contingent matter that this relation holds of the phases of changes, since, if

the temporal order is to be derived from the order of change, this would introduce contingency into the temporal order.

We have already seen that the relation *x* is potentially$_{\beta}$ *y*, where *x* and *y* are a boy and a man respectively, is asymmetric, but is it also transitive and connected? The worry here is that if the "*y*" in "*x* is potentially$_{\beta}$ *y*" is always the ultimate goal of the process, such as Coriscus-as-a-man, the relation will order the phases of the change relative to the goal but not relative to each other. In short, the answer is that *y* need not always be the goal, but to see this, it is useful to note that Aristotle makes a distinction between two types of necessity: absolute necessity and hypothetical necessity. Absolute necessity just means "incapable of being otherwise," in the sense, for instance, in which it is necessary for a human to have such and such characteristics (*Metaph*. 1006b31-2). Hypothetical necessity, by contrast, applies to things that must be if such and such a condition is to hold. Aristotle's most common examples of hypothetical necessity figure into answers to questions like "Why is it necessary for axes to be made of metal?" Answer: "If an axe is to chop wood, it must be made of a material that is suitably hard." But he also speaks of hypothetical necessity in connection with the phases of natural processes:

> Thus we should say, because man is an animal with such and such characters, therefore the process of his development is necessarily such as it is; and therefore it is accomplished in such and such an order, this part being formed first, that next, and so on in succession; and after a like fashion should we explain the construction of all other works of nature. (*PA* 640a33-b4)

> When an event takes place always or for the most part, it is not incidental or by chance. In natural products the sequence is invariable, if there is no impediment. (*Phys*. 199b24-6)

What I take this to mean is that if a boy is to develop into a man, the process of this development is necessarily "in such and such an order," because the actuality of posterior stages of the process presuppose the actuality of prior stages; that is, if the posterior stage is to be actual, it is necessary for the prior stage to be actual. Since the goal state of the change is to be accomplished "in such and such an order," it is clear that the relation ordering the phases of the change is meant to be asymmetric, connected and transitive since the phases apparently line up in a sort of hypothetical syllogism, i.e., if the actuality of the goal state presupposes the actuality of state *b*, and the actuality of state *b* presupposes the actuality of state *a*, then the actuality of the goal state presupposes the actuality of state *a*. It is also fairly clear that the relation *x* is potentially$_{\beta}$ *y* holds necessarily of phases of a change if the domain of *x* and *y* is human beings. Aristotle thinks that if *x* is a human, then it necessarily has the nature and characteristics of a human being (*Metaph*. 1006b28-34; *PA* 642a34-5), which will either be manhood for a man, or the potential for manhood and the motive force to realize it for a boy. Thus, if *x* is a human, and it is undergoing its natural development, it cannot fail to have manhood as the goal of this development, and if manhood is the goal of his development, then the order of his development must necessarily be thus and such.

1.4 Broadening the account

Even if the stages in a boy's development do, in fact, play out over time, the relation of prior and posterior in these stages may be described without an explicit reference to a temporal order: the prior is simply the necessary condition of the posterior, just as the fitness of an axe for chopping presupposes that it is made out of metal. Thus, Aristotle's metaphysical account of the development of natural substances such as humans gives us the resources to talk about the change from a boy to a man in abstraction from time. But what about change as such? In other words, can we generalize from the case of the boy growing into the man to say that any change whatever is ordered by the *x* is potentially$_{\beta}$ *y* relation? I will argue that it can, with some caveats, which I will come to shortly. The key point is that Aristotle thinks that everything that exists, at least in the sublunary realm, possesses an innate impulse of change that drives it toward some definite end or ends. Sometimes the end will be simple and immediate, like "being at the center" for earth. Sometimes the end will be complex and remote, like "being a man" for a boy or "being εὐδαίμων" for a man. Finally, sometimes the end will be consciously intended, like the possession of a cloak, or the taking of a walk in the case of a man, or it will be determined unconsciously and automatically, like "being at the center" in the case of a lump of earth.

1.4.1 Natural Substances and Artifacts

A strong indication that Aristotle takes all change to be teleological is his insistence in *Physics* book 2, chapter 1 (*Phys*. 192b9-23) that the category of

substance can be partitioned into two classes: natural substances,[9] and artificial creations or artifacts. Natural substances include "animals and their parts …, and the plants and the simple bodies (earth, air, fire, and water)." The criterion for this division is that natural substances possess a nature, that is, "a principle of change and of rest (in respect of place, or of growth and decrease, or by way of alteration)" while artifacts *qua* artifacts do not. Nonetheless, artifacts, *qua* the natural substances that they are made of, *do* have an "innate impulse of change" (*Phys*. 192b18-19: ὁρμὴν μεταβολῆς ἔμφυτον). For example, Aristotle alleges that the wooden bed might sprout a tree, or the clay jug might fall when dropped, reflecting the "innate impulse" of its material.

1.4.2 The lifeless and the living

1.4.2.1 The lifeless

The invariable tendency of earthy materials such as clay to move downward reflects the fact, in Aristotle's view, that the four elements are necessarily impelled toward a single characteristic end. Aristotle tells us in *De caelo* book 1, chapter 2, that "there is one sort of motion natural to each of the simple bodies," including the aether (*Cael*. 269a8-9). Thus, while we saw at *Metaphysics* 1048b37-1049a18 that earth does not possess an innate impulse of change that will transform it into a man, it *does* have the potentiality to be at the center of the universe and an innate impulse of change that drives it there, if nothing hinders it. Likewise, fire has the potentiality to be at the periphery of the universe, and will realize this potentiality if nothing hinders it. In *De caelo* 1.2, this point is

generalized for compounds of elemental bodies, whose natures, and therefore, whose "innate impulses," are taken to be simple functions of their elemental compositions:

> Bodies are either simple or compounded of such; and by simple bodies I mean those which possess a principle of motion in their own nature, such as fire and earth with their kinds, and whatever is akin to them. Necessarily, then, motions also will be either simple or in some sort compound—simple in the case of the simple bodies, compound in that of the composite—and in the latter case the change will be that of the simple body which prevails in the composition. (*Cael.* 268b26-a2)

At *Physics* 255a30-b31, Aristotle distinguishes the potentiality for air to perfect its nature by moving "high up" from the potentiality of water to acquire such a nature by being transformed into air, and this appears to be directly analogous to the distinction between the potentiality$_{\beta}$ for a boy to perfect its nature by becoming a man and the potentiality$_{\alpha}$ of a "seed" to acquire such a nature by being deposited in the uterus and "undergoing a change." Hence, the potentiality for air to perfect its nature by moving "high up" appears to be a potentiality$_{\beta}$, and assuming we can generalize this to earth and fire and compounds of earth, air, fire, and water (viz., to all sublunary inorganic bodies), then the phases of all natural changes involving these bodies are ordered by the asymmetric *x* is potentially$_{\beta}$ *y* relation.

Now one might think that a problem arises since at *Posterior Analytics* 94b35-95a9, Aristotle says the falling of a stone is due to necessity in the brute sense of necessity (i.e., "incapable of being otherwise") instead of in the sense of hypothetical necessity. Recall that hypothetical necessity is intimately bound up with teleology in that it relates goal states to their necessary conditions, and we

saw at *Parts of Animals* 640a33-b4 that, in this capacity, hypothetical necessity is indispensable in ordering the developmental phases of a living organism. But if Aristotle means, in these passages, to deny that hypothetical necessity applies to simple bodies, how can he claim that certain phases in the motion of these bodies are necessarily prior to others?

We need not assume, however, that Aristotle means to deny that hypothetical necessity applies to simple bodies, since he makes it clear in the immediately preceding passage, at *Posterior Analytics* 94b27-34, and also at *Parts of Animals* 642a1-b2, that teleology and brute necessity are not mutually exclusive. Moreover, it is clear that earth and fire *do* have characteristic tendencies to move toward a certain goal, and Aristotle even says that these tendencies are like desires (*Phys*. 192a16-25) and that change itself is like life (*Phys*. 250b14). In fact, Aristotle claims in *De caelo* book 3 that movement for the elements is movement towards form — earth is only properly earth when at rest in its natural place.

Perhaps Aristotle's idea is that a lump of matter is not formally complex enough to determine the order, as well as the goal of its local motions. Perhaps the order of the phases will be determined, instead, by the structure of the spatial continuum through which the elemental body moves. Suppose, for instance, that we have Owen's linear magnitude from A to D, and say that A is higher up than D, and that a lump of earth moves from higher to lower positions according to its nature. Suppose also that the line segment A-D is bisected at C and the line segment A-C is bisected at B. I claim that because of the structure of the

magnitude A-B-C-D, we must say that if the posterior stage of being at D is to be actual, it is necessary for the prior stage of being at C to be actual. But since intermediate position B is "higher up" than intermediate position C, in order for the posterior stage of being at C to be actual, it is also necessary for the prior stage of being at intermediate position B to be actual. So in order for the posterior stage of being at D to be actual, it is necessary for the prior stage of being at B to be actual. And if we assume that the magnitude A-B-C-D is a continuum, a similar necessary conditional will hold between any two distinct points along the line. Thus, as in the case of the development of the boy into a man, we can specify an order of the points on A-B-C-D with the relation *x* is potentially$_{\beta}$ *y*, where *x* and *y* range over this-lump-of-earth-at-position-A, this-lump-of-earth-at-position-B, etc.

1.4.2.2 The living

Aristotle also tells us, at *Metaphysics* 1047b35-1048a21, that living beings which possess only irrational potencies must act in a single characteristic way when the appropriate stimulus is present, much like a lump of earth when a hindrance to its falling is removed. Living beings, however, whose innate impulse of change is guided by a λόγος, have the ability to choose multiple, and often mutually exclusive ends, and this choice is determined by either rational desire (προαίρεσις) or irrational desire (ὄρεξις). But provided that some rational or irrational desire is dominant, action seems to follow necessarily:

> For whichever of two things the animal desires decisively (κυρίως), it will do, when it is present, and meets the passive object, in the way appropriate to the potency in question. Therefore everything which has a

> rational potency, when it desires that for which it has a potency and in the circumstances in which it has the potency, must do this. (*Metaph.* 1048a11-5)

The discussions of what commentators call the practical syllogism at *Nicomachean Ethics* 1147a24-31 and *De motu animalium* 702b20-25 confirms this point — action follows necessarily when a particular desire (either rational or irrational) becomes dominant and when there is an opportunity to act. In fact, since the act *is* the conclusion of the practical syllogism, Aristotle appears to assimilate the necessity of natural phenomena to the necessity of logical entailment:

> I need a covering, a coat is a covering: I need a coat. What I need I ought to make, I need a coat: I make a coat. And the conclusion I must make a coat is an action. And the action goes back to the beginning or first step. If there is to be a coat, one must first have B, and if B then A, so one gets A to begin with. Now that the action is the conclusion is clear. But the premises of action are of two kinds, of the good and of the possible. (*Mot.* 701a17-25)

Aristotle did not maintain a sharp distinction between the practical syllogism *qua* explanatory model and *qua* an actual piece of practical deliberation. In *De motu animalium*, it is used to explain action in animals as well as to literally describe pieces of rational deliberation in humans. These shifts between literal description and explanation are too obvious to be mistakes. It seems more likely that the practical syllogism is a very general psychological model whose broad applicability is based on an assumed continuity between ratiocination and natural phenomena. Indeed, for Aristotle, ratiocination is a natural phenomenon that can be used to explain itself.

The "premises" of the action are its goals, and goals may be either "of the good" or "of the possible," where the good is the very general objective of procuring clothing (or, more basically, warmth), and the possible is the proximate goal of obtaining a coat, or the means to make a coat. Notice the parallel here, between rational deliberation, and the unconscious, goal-directed change of a boy developing into a man: If the person is to get a covering, he must go about it "in such and such an order," because the actuality of posterior stages of the process of making a coat presuppose the actuality of prior stages; that is, if the posterior stage is to be actual, it is necessary for the prior stage to be actual. The point is not that everyone who wishes to procure a coat must go about it in the same way, but rather that any of the ways that one might go about it have a certain irreversible order (one cannot weave the fabric before shearing the sheep, or picking the cotton, for instance). Hence *x* is potentially$_{\beta}$ *y* holds of the phases of the coat's production, whatever the method of production.

This relation holds, in fact, because the form of the finished coat and the motive force to make it are in the mind of the tailor. In an analogous case at *Metaphysics* 1048b37-1049a18, building materials are said to be potentially a house in the sense of potentiality$_{\beta}$ since the form and the motive force to realize it are in the mind of the builder. Thus the relation *x* is potentially$_{\beta}$ *y* holds of the phases of the change as long as "the agent has willed it" and "if nothing external hinders:"

> The delimiting mark of that which as a result of thought comes to exist in complete reality from having existed potentially is that if the agent has willed it, it comes to pass if nothing external hinders, while the condition on the other side—viz. in that which is healed—is that nothing in it

> hinders the result. It is on similar terms that we have what is potentially a house; if nothing in the thing acted on—i.e. in the matter-prevents it from becoming a house, and if there is nothing which must be added or taken away or changed, this is potentially a house; and the same is true of all other things the source of whose becoming is external. (*Metaph.* 1049a5-12)

Of course, the ability to choose multiple, and often mutually exclusive ends will make the goal-directed activities of animals more complex than that of inorganic substances. Nonetheless, Aristotle thinks that humans, for instance, still have an overarching single goal that is characteristic of the type of natural substance that they are, viz., happiness (εὐδαιμονία). And although many may mistake it for an apparent good like wealth or honor, happiness is, in fact, an active life of the soul, and in particular, an active life of its rational principle in accordance with excellence (*EN* 1.7).

This raises the problem of how to work out an account of the relationship between general and specific goals for an organism. Waterlow suggests that specific and general goals of an organism may be related on the analogy of matter to form, and this seems plausible and Aristotelian.[10] But what is required for the purpose at hand, i.e., for the relation *x* is potentially$_{\beta}$ *y* to hold necessarily of the phases of a change of a biological organism, is merely that all of the actions and modifications of such an organism are characteristically goal-directed, regardless of how they are inter-related. If every action or modification of a biological organism can be explained in terms of a goal (whether complex or simple, immediate or general, conscious or unconscious) then the order of the prior and posterior in change can be determined by this teleology, combined, perhaps, with certain aspects of the medium in which the change takes place (e.g., the structure

of the space, magnitude, or range of qualities, which the changing body must traverse).

Aristotle assures us, quite generally, that "nature does nothing without reason or in vain" (*Cael.* 291b13; cf. *GA* 744a36, b16; *PA* 686a22), but he clearly thinks his strongest case for teleology is in the biological realm. In *De anima* 2.2 and 2.3 Aristotle tells us that living things are distinguished from the lifeless by the possession of a soul, and souls are defined and classified in terms of certain characteristic developmental and behavioral potentialities. Even plants have souls which have the goal of nutrition and growth, and this is the common denominator of all biological teleology. Since animals do not find their food automatically in the soil as plants do, their souls also have the functions of motility and sensitivity; sensitivity to identify food sources, and motility to go and acquire them. Finally, the soul of the human has the further function of reasoning well, or acting in accordance with a rational principle, and this serves, as I have said, the characteristically human goal of happiness or εὐδαιμονία.

Hence, we may extend our earlier conclusion about humans to apply, in the sublunary realm, at least, to natural substances as well as to artifacts (insofar as they are made of natural substances), to the living as well as to the inorganic, and to simple as well as compound bodies. Since Aristotle thinks that (a) all the activities of these natural substances are characteristically goal-directed, and that (b) if something is a natural substance of a certain sort, then it necessarily has the nature and characteristics of that sort of substance, we may conclude that (i) if something is one of these natural substances (or an artifact composed of them),

and it is engaged in a natural change, it cannot fail to have *some* specific goal as the goal of this change, and (ii) if *x* and *y* are phases in a natural change of one of these natural substances, then the order of the phases of this change must necessarily be ordered by the relation *x* is potentially$_{\beta}$ *y*. Hence, the relation *x* is potentially$_{\beta}$ *y* must hold necessarily of the phases of a change of such a substance in the sense that if *x* and *y* are distinct phases in one of its changes, these phases must necessarily be related by this relation.

1.5 The definition of change

This is consistent with the fact that that the definition of change in *Physics* book 3, chapter 1 ("change is the actuality of what exists potentially$_{\beta}$, *qua* such [viz., existing potentially$_{\beta}$]" (*Phys*. 201a10-1)) is offered as a general definition of change, and not simply as a definition of the process of maturation for biological organisms. It is also consistent with the fact that Aristotle holds at least sublunary change to be always change toward a particular terminus of some definite type (i.e., according to one of the categories; see *Physics* 200b33-201a8). He argues for this point at *Physics* 241a26-b12, in the process of establishing the claim that "no process of change is infinite:"

> Our next point is that no process of change is infinite: for every change, whether between contradictories or between contraries, is a change from something to something. Thus in contradictory changes the positive or the negative, as the case may be, is the limit, e.g. being is the limit of coming to be and not-being is the limit of ceasing to be: and in contrary changes the particular contraries are the limits, since these are the extreme points of any such process of change, and consequently of every process of alteration: for alteration is always dependent upon some contraries.

> Similarly contraries are the extreme points of processes of increase and decrease: the limit of increase is to be found in the complete magnitude proper to the peculiar nature of the thing that is increasing, while the limit of decrease is the complete loss of such magnitude. (*Phys*. 241a26-b2)

Aristotle goes on to argue that locomotion is always finite, even though it is not always between contraries. Locomotion must come to an end, he argues, since "it is inconceivable that that which cannot complete a change should be in process of changing to that to which it cannot complete a change." Although this argument begs the question, by assuming that every locomotion must be locomotion to some definite end, it illustrates Aristotle's commitment to this principle, at least in the sublunary realm.[11] An important exception to this is the circular motion of the heavens, which is infinite precisely because it does *not* take place between contrary termini, at least in any normal sense of contrariety. The circular motion of the heavens poses a special challenge to the interpretation I have been developing, and I will take this up in Section 1.7. But first, I need to address the relationship between my distinction of potentiality$_\alpha$ and potentiality$_\beta$ and a more familiar distinction Aristotle makes in *De anima* 2.5.

1.6 Potentiality$_\alpha$ and potentiality$_\beta$ vs. first and second potentiality

In *De anima* 2.5, Aristotle makes a distinction between two types of potentiality, which he does not name, but which the ancient commentators call "first" and "second" potentiality. This distinction is related to, but is not identical to the distinction that I have made between potentiality$_\alpha$ and potentiality$_\beta$. The best way to explain the relation between these distinctions is to use an illustration, but as a

preliminary, let us first recognize another distinction implicit in *De anima* 2.5, viz., between an actuality of a potentiality and an actualization of a potentiality. In English, "actuality" denotes the state of being actual while "actualization" denotes the change of state of becoming actual. In the Greek of *De anima* 2.5, the state of being actual is denoted in a number of ways: with the nouns ἐντελέχεια and ἐνέργεια,[12] with the verb "to be" combined with the dative of manner ἐντελεχείᾳ and ἐνεργείᾳ,[13] and with the articular infinitive τὸ ἐνεργεῖν.[14] The change of state of becoming actual is called an ἐπίδοσις εἰς ἐντελέχειαν at 417b6-7, and is also indicated by the phrases εἰς ἐντελέχειαν ἄγειν (417b10) and ἐνεργείᾳ γίνονται *x* (417a30), where *x* is the sort of thing that the subject becomes ἐνεργείᾳ. Thus, the actuality (ἐντελέχεια or ἐνέργεια) of a potentiality to φ is the act of φ-ing, while the actualization (ἐπίδοσις εἰς ἐντελέχειαν) of a potentiality to φ is the coming to be of the act of φ-ing.

For our example, let us suppose that to φ is to know how to build a house. (This is one of Aristotle's examples, at *De anima* 417b9.) The potentiality corresponding to this actuality, then, is the condition of being able to know how to build a house, and the actualization of this potentiality is the process of learning how to build a house (i.e., the coming to be of the state of knowing). The condition of being able to know how to build a house (or the ability to learn housebuilding) is what the ancient commentators call first potentiality. The actuality of this potentiality, i.e., knowing how to build a house, is called a first actuality. But once a person knows how to build a house, he has the ability to go out and start building, unless something hinders him. Hence, this ability, i.e.,

knowing how to build a house, which we have already identified as the first actuality of the first potentiality of being educable in the art of housebuilding, is also a potentiality, because to know how to build a house represents the potentiality to go out and start building a house. The ancient commentators call this second potentiality. The actuality corresponding to this potentiality, i.e., the activity of housebuilding, is called a second actuality, and it is actualized in the coming to be of the process of building a house.

De anima 2.5 tells us that the first potentiality consisting in the ability to know how to build a house is present because its subject, or the thing that has the potentiality, is a certain sort of thing, or as Aristotle says, because "its kind is such and such and its matter" (*DA* 2.5 417a27). Now there seems to be two ways to interpret the phrase "and its matter": either as a genuine conjunct, or as an epexegesis. If the former, then a first potentiality exists in a subject because the subject is a member of a certain natural kind, *and* because its matter is such and such. If the latter, then the point is just that a subject has a first potentiality because it is a member of a certain natural kind, and that natural kinds bear a kind of similarity to matter. In favor of the latter reading is the fact that Aristotle *does* say, in at least three passages in the *Metaphysics*, that the kind is the matter of the species, and in each case, the point of identifying kind with matter seems to be metaphorical.[15] At *Metaphysics* 1038a6-7, Aristotle seems to argue that genera do not exist independently of species, just as matter does not exist independently of form. Likewise, at *Metaphysics* 1024b8-9, where he says that a genus is matter in the sense of a "substratum … to which differentia or qualities belong," he

seems to be arguing that just as qualities are predicated of matter, differentiae are predicated of genera.

Yet the examples used in these two instances can easily be read to refer to matter as a *particular* substrate rather than as a universal one. At *Metaphysics* 1024b8-9, Aristotle mentions genera "in the sense in which plane is the kind of plane figures and solid of solids," and following Mueller, it is possible to take Aristotle to understand points, lines, surfaces and solids to be the intelligible matter of *particular* geometrical objects, i.e., as a sort of indeterminate particular substratum upon which the properties of individual geometrical objects are imposed.[16] Similarly, at *Metaphysics* 1038a6-7, "voiced sound" or φωνή, which is said to be the genus and matter of types of sounds, could be taken as the universal substrate of these types, as at *Metaphysics* 998a20-5, or as their particular substrate and what they are made of, as at *Generation of Animals* 786b21.

In the light of this, it seems that one *could* make the argument that Aristotle refers indiscriminately to both universal and particular substrata when he refers to "genus and matter," and that what is meant at *De anima* 2.5 417a27 is that a first potentiality exists in a subject because the subject is a member of a certain natural kind, *and* because its matter is of a certain kind. This is basically the position taken by Philoponus and Simplicius,[17] who interpret Aristotle to be claiming that to have a first potentiality is to be a substratum, either particular or universal, that is receptive to the relevant form or quality specified in the potentiality. What I take this to mean is that humans, for instance, are able to

learn how to build houses because this is characteristic of them as a species, which essentially means that their minds are of a sort that can receive a certain formal content. But equally, earth and "seed" are able to become the flesh and bones of an animal, because they are capable of receiving the formal structure of such an organism. *If* one takes this reading of *De anima* 2.5 417a27, then what I have called potentiality$_{\alpha}$, insofar as it represents the bare fitness to be endowed with a form or quality, corresponds to first potentiality.

Now insofar as potentiality$_{\beta}$ is the potentiality of a thing, once endowed with a form, to perfect that form, it does resemble second potentiality, since second potentiality is the potential, once endowed with the knowledge of housebuilding, for instance, to make it actual by going out and building a house. There is a problem, however, partly because in *De anima* 2.5, the focus is on the housebuilder instead of on the house, and partly because in *De anima* 2.5, using one's skill to build a house (*DA* 417b9) is thought of as an example on all fours with using one's wisdom, and merely contemplating a bit of knowledge. Like the ability to contemplate the art of housebuilding, the ability to engage in the activity of housebuilding is thought, in *De anima* 2.5, to be fulfilled instantaneously, as soon as the activity commences. But clearly, if potentiality$_{\beta}$ were only the potentiality of a thing to *start* perfecting a form that it is endowed with, it could not order the phases of the process of perfecting this form. So potentiality$_{\beta}$ is not the same as second potentiality.

To say that that the activity of housebuilding is achieved as soon as it commences is to treat it as an ἐνέργεια in the sense in which ἐνέργειαι are

defined in contradistinction to κινήσεις in *Metaphysics* book Θ chapter 6. The problem, however, is that housebuilding is adduced as an example of a κίνησις in *Metaphysics* book Θ chapter 6 1048b29-31. *Metaphysics* book Θ chapter 6 claims that an ἐνέργεια is characteristically complete, and the test of this is being able to say, at the same time, where φ-ing is the relevant activity, both that one is φ-ing and that one has φ-ed. Aristotle claims that everything that is moving has moved in *Physics* 6.6, but the point made in *Metaphysics* book Θ chapter 6 is different. In *Physics* 6.6, Aristotle says this to make the point that in continuous motion, there is no first moment of being away from an origin. In *Metaphysics* book Θ chapter 6, however, he is making the point that certain activities always have the perfect verb tense because they always represent a complete achievement. One can say that one sees and, at the same time, has seen, or thinks, and at the same time, has thought, because the activities of seeing and thinking are achieved as soon as they commence. But *Metaphysics* book Θ chapter 6 implies that in the case of housebuilding, one cannot do the same because at the time when one starts building, one has not finished building a house yet.

In *De anima* 2.5, however, Aristotle grants himself the license to speak loosely: "First, let us speak as though τὸ πάσχειν, τὸ κινεῖσθαι, and τὸ ἐνεργεῖν are the same thing; for κίνησις is a sort of ἐνέργεια, although an incomplete one, as has been said elsewhere" (*DA* 2.5 417a14-17). Since κίνησις is a sort of ἐνέργεια, in other words, we may speak as though τὸ κινεῖσθαι is the same as τὸ ἐνεργεῖν, *when it is not.* Aristotle is telling us that, to achieve a certain level of generality, we can, for the moment, ignore the distinction between

agent and patient *and* the fact that a κίνησις is incomplete. This does no harm in *De anima* 2.5 because, there, the focus is on the potentiality of the agent to build, which is, in a sense, fulfilled as soon as he starts building. But if we consider the object of the housebuilding, i.e., the bricks that get built into a house, the unachieved τέλος, and therefore, the incompleteness of the process of housebuilding comes sharply into view, and this requires a further analysis. That further analysis, I believe, is found in *Physics* 3.1-3.

1.6.1 The further analysis

An influential interpretation of Aristotle's definition of motion in *Physics* 3.1 is that of L. A. Kosman.[18] Kosman argues that Aristotle's characterization of the motion of housebuilding, for instance, as an incomplete actuality, or an actuality that exists "*qua* potentiality," viz., *qua* still potentially and not yet actually a house, requires a distinction between differing degrees of potentiality much like the distinction between first and second potentiality. Kosman makes the distinction between what he calls "potentiality$_1$" and "potentiality$_2$" as follows: The potentiality$_1$ of bricks to be a house is a double potentiality to be a house, or the potentiality to be potentially a house. The idea is that, when bricks are being built into a house, or are part of some partially constructed house, their potential to be a house is more actual than when they are lying in a heap on the ground. Kosman expresses the etiolated sense of being potentially a house (when the bricks are lying in a heap) by the notion of a double potentiality, so that what is potentially potentially a house (i.e., what is "potentiality$_1$" a house) is less actual

than what is actually potentially a house. What is actually potentially a house is in motion toward being a house, but is not yet a house. It is the "actuality of what exists potentially" (viz., an incomplete actuality of an incomplete potentiality), *"qua* existing potentially" (viz., *qua* still potentially and not yet actually a house). It is what Kosman calls "potentially$_2$" a house, to distinguish it from the double potentiality of what is potentiality$_1$ a house. Finally, "potentiality$_2$," or the actuality of what is potentially potentially a house, is also called an "actuality$_1$" to distinguish it from the actuality of what is actually actually a house (a fully constructed house), viz., an "actuality$_2$."

Note that the first potentiality of being able to know how to build a house is also a double potentiality, i.e., it is the potentiality to be a potential housebuilder in the sense of someone who knows how to build houses and can go out and do so, if nothing hinders him. In fact, Kosman's entire scheme of single and double potentialities fits perfectly with the distinction of first and second actuality: The first actuality of knowing how to build a house is the actuality of the double potentiality to be able to know how to build a house, and the actuality$_1$, of being in the process of becoming a house is the actuality of the double potentiality to be able to be in motion toward household. And just as the second potentiality of the knowledgeable housebuilder for the second actuality of building a house is identical to the first actuality of being a knowledgeable housebuilder, the potentiality$_2$ of the bricks in the process of becoming a house for the actuality$_2$ of being a house, is identical to the actuality$_1$ of being in the process of becoming a house. What this shows, I believe, is that, at a certain level of

abstraction, Kosman's scheme of single and double potentialities fits both the phenomenon of motion and the acquisition and exercise of a disposition. But, as Kosman demonstrates, when we consider the substance of what Aristotle is talking about in *Physics* 3.1-3 and *De anima* 2.5, i.e., when we start thinking φυσικῶς instead of λογικῶς,[19] as Aristotle would say, we see that we cannot identify potentiality$_1$ with first potentiality and potentiality$_2$ with second potentiality. The first thing Kosman points out is that the actualization of a first potentiality will differ from the actualization of a potentiality$_1$ in that the latter will happen instantaneously while the former will take time. This is because the actualization of the ability to learn how to build is the process of learning how to build, while the actualization of the potentiality$_1$ of the bricks to be in motion toward becoming a house is the inception of the process of building.[20] The problem is that in the scheme of *De anima* 2.5, the first actuality/second potentiality is a disposition, e.g., the disposition, once one has learned housebuilding, to build a house. In Kosman's interpretation of *Physics* 3.1, however, the actuality$_1$/potentiality$_2$ is the state of being in motion. Dispositions and motions are disanalogous in the way that they come about (the one takes time; the other does not).

Moreover, the second actuality of building needs, in any event, to be identified with what Kosman calls the actuality$_1$/potentiality$_2$ of the bricks being built. This is because Aristotle claims in *Physics* 3.3 that the process of housebuilding in the housebuilder is numerically identical, but different in definition, to the process of being built into a house undergone by the bricks.

This identification, I believe, is the key to understanding the relationship between *De anima* 2.5 and *Physics* 3.1. In *De anima* 2.5 Aristotle looks at housebuilding from the point of view of the housebuilder, and from this vantage, housebuilding is just an activity in which a knowledgeable housebuilder has the potentiality to engage. Hence, there is no need, in *De anima* 2.5, to worry about the incompleteness of this activity, since the incompleteness is in the partially built house. When we get to *Physics* 3.1, however, the focus is on the house, and so Aristotle gives us a further analysis of housebuilding, viz., an analysis which emphasizes the fact that housebuilding and being built into a house have the further goal of a completed house. This further analysis is possible because the activity of building is the same in substrate as the process of being built, though different in definition. So, there is no question of an identification of the distinction between first potentiality and second potentiality in *De anima* 2.5 with the distinction between potentiality$_1$ and potentiality$_2$. The latter is a new distinction intended to apply to processes, not to dispositions.

1.6.2 What about potentiality$_\alpha$ and potentiality$_\beta$?

But what about my distinction between potentiality$_\alpha$ and potentiality$_\beta$? How does it fit into all of this? To answer this, let us turn away from housebuilding and consider an example of a natural change. To keep it simple, let us consider the case of something becoming fire and then, *qua* fire, rising, which Aristotle explicitly assimilates to the *De anima* 2.5 scheme of first and second potentiality in *Physics* 8.4 255a30-b31. After recounting this scheme using the example of

the acquisition and exercise of knowledge, Aristotle goes on to say "In regard to natural bodies also the case is similar. Thus what is cold is potentially hot: then a change takes place and it is fire, and it burns, unless something prevents and hinders it" (*Phys*. 8.4 255a5-7). So the idea seems to be this: just as a human has the ability to know about housebuilding, a pile of kindling ("what is cold") has the ability to be fire (it is "potentially hot"). Each of these are first potentialities. The first potentiality to be fire is actualized by the ignition of the kindling. The first potentiality to be a knower about housebuilding is actualized by the process of learning how to build a house. In the passage under consideration, Aristotle goes on to the case of what is heavy being potentially light, but switches his example to water becoming air. Since this would equally apply to kindling becoming fire, however, let us stick to Aristotle's original example. Aristotle implies that there are two senses of being potentially light, viz., the first potentiality of the kindling, which is heavy, yet can become light if it is turned into fire by being burned, and a second potentiality of the fire which is still potentially light because it is prevented from actualizing its lightness by some impediment. So, the second potentiality/first actuality disposition of knowing about housebuilding corresponds to the second potentiality/first actuality disposition of being potentially light, but being prevented from actualizing one's lightness by some impediment.

So far so good, but then Aristotle's exposition becomes confusing. Based on *De anima* 2.5, he should say that the second actuality of the second potentiality/first actuality disposition of being potentially light, in the sense of

being prevented from actualizing one's lightness by some impediment, is the activity of *rising*. Then, a further analysis of the activity of rising, in the manner of Kosman's interpretation of *Physics* 3.1, would show that rising involves the further goal of "being high up." Instead, Aristotle confuses matters by saying both that the second actuality of the second potentiality/first actuality disposition of being light is "rising" (255b21) *and* that it is "being high up" (255b11). Armed with our analysis of housebuilding, however, we can fill in the lacunae. Just as the bricks lying in a heap are potentially$_1$ or potentially potentially a house, the fire when prevented from rising is potentially$_1$ or potentially potentially high up. And just as the bricks in a house under construction are potentially$_2$ or actually potentially a house, the fire, as it is rising, is potentially$_2$ or actually potentially high up, where the rising is the actuality of the potentiality to be high up, *qua* still potentially high up. Finally, just as the bricks, when incorporated in a fully constructed house are actually$_2$ a house, the fire, when high up, is actually$_2$ high up.

Notice that by switching from the example of a transitive activity like housebuilding to an intransitive activity like rising, not only is the second actuality the same in substratum, yet different in definition, as potentiality$_2$/actuality$_1$, but also the second potentiality/first actuality is the same in substratum, but different in definition, as the potentiality$_1$. In housebuilding, the knowledge of housebuilding (= second potentiality/first actuality) and the suitability of the bricks for building (= potentiality$_1$) were located in separate subjects. In the case of fire rising, however, the disposition to rise (= second

potentiality/first actuality) and the potential to be rising (= potentiality$_1$) are in the same subject. But it is the same thing to say that something has the disposition to rise and the potential to be rising, and since these potentialities are in the same subject, they are numerically identical as well.

Now the point of the distinction I found in *Metaphysics* 1048b37-1049a18 between potentiality$_\alpha$ and potentiality$_\beta$ was to distinguish between the bare fitness to be endowed with a nature (potentiality$_\alpha$) and the potentiality, once endowed with a nature, to perfect it (potentiality$_\beta$). I said, above, that if one interprets Aristotle to be claiming at *De anima* 2.5 417a27 that a thing has a first potentiality because the subject is a member of a certain natural kind, *and* because its matter is of a certain kind, one can identify first potentiality with potentiality$_\alpha$. As for potentiality$_\beta$, since it applies to the tendency of a natural substance to move toward a certain goal, while the tendency is both latent and active, potentiality$_\beta$ would appear to incorporate the second potentiality and potentiality$_1$ to rise as well as the potentiality$_2$, once rising, to be "high up."

1.7 Caveats and exceptions

An exception to the generalization that the phases of all sublunary natural changes are ordered by the asymmetric *x* is potentially$_\beta$ *y* relation, perhaps, is the case of an elemental transformation, such as when air changes to water, or vice versa. We have already seen from *Metaphysics* 994a22-994b3 quoted above in Section 1.1, that elemental transformations *are* reversible. But this just reflects the fact that there is no nature common to air and water that is capable of guiding the

transformation. When transforming in this way, elemental bodies are, in a sense, between natures, and thus have no single innate impulse of change. What they have throughout the transformation is a potentiality$_{\alpha}$, as matter, to bear properties and be endowed with this or that elemental nature, but since matter can indifferently bear any elemental nature, the relation *x* is potentially$_{\alpha}$ *y* is not asymmetric.

Another possible exception is the case of an accidental confluence of the changes of separate natural substances (for instance, one substance hindering or altering another's change). It seems to me that each of the actions involved in the confluence may be explained entirely in terms of individual teleologies. When a creditor and a debtor go to the market, for instance, and the creditor collects his debt, both the creditor and the debtor intended to go shopping, and this is "cause without qualification" of their actions (*Phys*. 2.5 197a8-15). But the fact of the confluence and its outcome, viz., the collection of the debt, has no cause, strictly speaking, and can only be accounted for as a coincidence. The problem, as I see it, is that it must be a spatio-*temporal* coincidence, if anything, and one must appeal to a temporal order to describe it.

The case of biological decay would seem to be another exception, since this type of change obviously proceeds from a more perfect state to a less perfect state, and it does not obviously have a goal. But I think that biological decay can be subsumed under accidental change. The idea is that while this type of decay might seem to be simple in nature, as just the falling apart of organized systems, it is actually the compound of at least two teleological motions. Take the

progressive deterioration of a biological organism due to ageing, for instance. In this case, decay is merely the losing battle fought by the organizing life force of the organism against the reassertion of the natural teleologies of its material body. The old man stoops to the ground, for instance, because the downward-tending earthy components of his body are overpowering the vital force exerted by his muscles to keep him upright. The case of non-biological decay is even easier to accommodate. Consider artifacts, for instance, like the temples of ancient Greece. Once they were endowed with the form of a temple and stood upright. But now they lie on the ground in ruins because they were composed of stone — an earthy material with a downward-tending drive, and a drive that meets with no active opposition since, unlike the old man, a temple has no innate teleological impulse to hold the stones in place.

1.7.1 The problem of celestial motion

A third, and probably more serious exception may be the case of celestial motion.[21] I have claimed that *x* is potentially$_{\beta}$ *y* orders the phases of a natural change in the sublunary realm according to the degree to which these phases attain a τέλος. A τέλος is a goal as well as a formal perfection, and insofar as phases of a change fall short of a τέλος, they are formally incomplete and imperfect. But consider the circular motion of the heavens. It is a natural motion, because circular motion is the natural motion of the fifth element, of which the celestial spheres are made (*Cael.* 269a5-7, b2-5, 272a5-7, 289a15-6). Each of the simple bodies, including the fifth element, has a nature, and, therefore, a motion

that is natural to it (*Cael*. 268b14-6, 269a8-9), and indeed, there is no movement without natural bodies (*Cael*. 279a15-6). The eternality of the celestial rotations, in fact, depends on their being natural (*Cael*. 286a16-8). But the natural motion of the heavens, at first sight, does not seem to be teleological. In *Physics* 8.9 Aristotle distinguishes the circular motion of the heavens from sublunary rectilinear motion as follows:

> In rectilinear motion we have a definite beginning, end (τέλος), and middle which all have their place in it in such a way that there is a point from which that which is in motion can be said to start and a point at which it can be said to finish its course (for when anything is at the limits of its course, whether at the whence (ὅθεν) or at the whither (οὗ), it must be in a state of rest). On the other hand a circular motion is unbounded. (*Phys*. 265a29-32)

In *De caelo* 2.1, he says that the circular motion of the heavens has "no beginning or end (τελευτήν), but [is] unceasing through the infinity of time" (*Cael*. 284a8-10), and in *De caelo* 2.6 he says that circular locomotion, having no beginning or limit or middle *simpliciter* (ἁπλῶς), has neither whence (ὅθεν) nor whither (οἶ) nor middle: for in time it is eternal, and in length it returns upon itself without a break" (*Cael*. 288a22-4).

This certainly seems like a denial that celestial motion is teleological, which should disqualify the relation *x* is potentially$_{\beta}$ *y* from ordering its phases, and deprive Aristotle of the means to define the order of celestial motion in abstraction from the order of time. I could claim that Aristotle's theory of the order of change is essentially a theory of the order of *sublunary* change that just fails to account for celestial motion, but since Aristotle claims that the order of time is derived from the order of change without qualification, this would saddle

Aristotle with a philosophical mistake. And since my original intention was to acquit Aristotle of such a mistake, albeit a different one, I would just be trading one problem for another. Indeed, *this* mistake of failing to account for the order of celestial change might seem more serious, since, although celestial motions are not explicitly invoked in *Physics* 4.11 where Aristotle derives the order of time from the order of change, Aristotle *does* give them very critical roles in his overall theory of time. In *Physics* 4.14, he tells us that uniform circular motion is above all the measure of motion because it is best known, and that it is the measure of all other motions as well as time (*Phys*. 223b12-21). We are also told, later in the *Physics*, that celestial motion is the primary motion because it is the measure of all motions, and it is the measure of all motions because it is the primary motion (*Phys*. 265a13, b8-9). Finally, at *Generation and Corruption* 337a22-33, Aristotle seems to suggest that since only the circular motion of the heavens is "continuous with itself" (i.e., the "in which" and the subject of change is eternally constant), it alone can ensure the eternal continuity of time.

1.7.2 Is the natural motion of the heavens an ἐνέργεια?

An obvious explanation for the apparent lack of teleology in the circular motion of the heavens would seem to be that it is an activity (ἐνέργεια) instead of a process (κίνησις), in the sense in which these two terms are distinguished in *Metaphysics* book Θ chapter 6. The criteria for ἐνέργειαι laid out in *Metaphysics* 1048b18-36 make it clear that ἐνέργειαι are not teleological in the sense that κινήσεις are, and celestial motions seem to meet a number of these criteria. First,

we are told that κινήσεις have limits while ἐνέργειαι have none. And in *Physics* 8.9, we are told that rectilinear κίνησις is bounded and has limits which are its beginning and end but κίνησις along a circumference is unbounded and has no limits which are its beginning and end (*Phys*. 265a27-32). Second, *Metaphysics* book Θ chapter 6 tells us that actions which are ends in themselves, or in which an end is present are ἐνέργειαι, otherwise, they are κινήσεις (*Metaph*. 1048b18-36). If limits are to be identified with ἀρχαί and τέλη, as *Physics* 265a27-32 seems to suggest, then *De caelo* 2.1 seems to imply that celestial motions have no ἀρχαί or τέλη, and themselves are ἀρχαί and τέλη, of other motions:

> There is some immortal and divine thing which possesses movement, but movement such as has no limit and is rather itself the limit of all other movement. A limit is a thing which contains; and this motion, being complete, contains those incomplete motions which have a limit and a stopping point, having itself no beginning or end, but unceasing through the infinity of time, and of other movements, to some the cause of their beginning, to others offering the stopping point." (*Cael*. 284a3-11)

Third, *Metaphysics* book Θ chapter 6 tells us that since κινήσεις have limits while ἐνέργειαι have none, κινήσεις must cease while ἐνέργειαι need not (*Metaph*. 1048b26-7). *Physics* 8.8 tells us that since circular celestial motion has no limit, it is the only locomotion that need never cease (*Phys*. 265a10-12). Fourth, *Metaphysics* book Θ chapter 6 tells us that ἐνέργειαι are complete (τέλειος) while κινήσεις are not. Both the *Physics* and *De caelo* tell us that circular locomotion is complete (τέλειος; *Phys*. 264b28, 265a17, *Cael*. 284a7). Finally, *Metaphysics* book Θ chapter 6 tells us that if to φ is an ἐνέργεια, then saying that something is φ-ing must always be compatible with saying, at the same time, that it has φ-ed, where having φ-ed represents a complete

achievement.[22] And *Physics* 8.9 implies that the circular motion of the heavens always meets this test because a revolving celestial body is always "in its end" (ἐν τέλει; *Phys.* 265a32-b1), i.e., at every point the celestial body "has completed" a circuit.

Another sign that Aristotle takes circular motions of the heavens to be ἐνέργειαι seems to be that he uses the words ἐνεργείᾳ and ἐνεργεῖν to describe the way in which the celestial spheres and their motions exist, and that he denies that they exist potentially. At *Metaphysics* 1050b18, Aristotle says that imperishable things exist ἐνεργείᾳ, and then proceeds to say:

> Nor does eternal movement, if there be such, exist potentially; and, if there is an eternal mobile, it is not in motion in virtue of a potentiality, except in respect of "somewhence" (ποθέν) and "somewhither" (ποί) (there is nothing to prevent its having matter which makes it capable of movement in various directions). And so the sun and the stars and the whole heaven always act (ἀεὶ ἐνεργεῖ), and there is no fear that they may sometime stand still, as the natural philosophers fear they may. Nor do they tire in this activity; for movement is not for them, as it is for perishable things, connected with the potentiality for opposites, so that the continuity of the movement should be laborious; for it is that kind of substance which is matter and potency, not actuality, that causes this. (*Metaph.* 1050 b20-8)

1.7.3 Possible revision of the definition of "x is potentially$_\beta$ y"

Note that in this last passage, Aristotle does not deny *all* potentiality to celestial motion, but, rather, he claims that the "eternal mobile [is] not in motion in virtue of a potentiality, except in respect of 'somewhence' and 'somewhither' (there is nothing to prevent its having matter which makes it capable of movement in various directions)." A few lines before this passage, in fact, he says that "there is

nothing to prevent [imperishable things] being [potentially existent] in some respect, e.g. potentially of a certain quality or in a certain place" (*Metaph.* 1050b16-8). At *Metaphysics* 1069b24-6, he says, "Now all things that change have matter, but different matter; and of eternal things those which are not generable but are movable in space have matter—not matter for generation, however, but for motion in respect of 'somewhence' and 'somewhither,'" and at *Metaphysics* 1044b6-8, "But in the case of natural but eternal substances another account must be given. For perhaps some have no matter, or not matter of this sort but only such as can be moved in respect of place." (cf. *Metaph.* 1042b5-6)

What is a potentiality for motion in respect of "somewhence" and "somewhither?" Since "somewhence" (ποθέν) and "somewhither" (ποί) are the indefinite forms of "whence" (ὅθεν) and "whither" (οἷ), which Aristotle uses to refer to the termini of rectilinear movement (e.g., *Cael.* 288a22-4), perhaps it is best to think of the potentiality to move in respect of "somewhence" and "somewhither" as a potentiality to move non-teleologically, i.e., a potentiality for merely moving in some unspecified direction or of being at unspecified different places at different instants. Perhaps this sort of potentiality — let us call it "potentiality$_\gamma$" — can be used to define a new relation *x* is potentially$_\gamma$ *y*, which can order the phases of circular celestial motions. In this case, we can rely on Aristotle's claim that "no potential relates to being in the past, but always to being in the present or future" (*Cael.* 1.12 283b13-14), to secure the required asymmetry. There is, of course, a circularity here, if we define potentiality$_\gamma$ in terms of past, present, and future, but perhaps not a vicious circularity, if we can

restrict the relation to hold only between phases that are sufficiently close in the kinetic sequence.[23] And one might even take the position that definitions, quite generally, cannot avoid being circular in some ultimate sense, and that our objective is just to avoid circularities that are obvious or vicious.

What, then, is the relationship between potentiality$_\beta$ and potentiality$_\gamma$? If we identify first potentiality with potentiality$_\alpha$, and take potentiality$_\beta$ to incorporate second potentiality as well as Kosman's potentiality$_1$ and potentiality$_2$, as I suggested in Section 1.6, then potentially$_\gamma$ will incorporate nothing like potentiality$_\alpha$, but only part of what I call potentiality$_\beta$. Kindling, for instance, has the first potentiality or potentiality$_\alpha$ to transform into fire, but the fifth element does not have the potentiality to transform into any of the other elements. So the fifth element has no first potentiality or potentiality$_\alpha$. Fire has the second potentiality/first actuality disposition of being potentially light, when it is prevented from actualizing its lightness by some impediment. The actuality of this disposition (i.e., its second actuality) is the activity of rising. But the potentiality of the fifth element to move "in respect of 'somewhence' and 'somewhither'" is never impeded (*Cael*. 279a33-5), so it is not quite the same as the second potentiality/first actuality disposition of being potentially light.

The activity of rising, in the case of fire, can be further analyzed in the manner of Kosman's interpretation of *Physics* 3.1, in terms of the goal of being "high up" (viz., the fire when prevented from rising is potentially$_1$ or potentially potentially high up; the fire, as it is rising, is potentially$_2$ or actually potentially high up; the fire, when high up, is actually$_2$ or actually actually high up). But the

movement of the fifth element "in respect of 'somewhence' and 'somewhither'" cannot be further analyzed in terms of the ultimate τέλος of being in a natural place like "high up" because the fifth element, unlike fire, is already in its natural place when it is moving, and, indeed, cannot change its place in this sense (*Cael.* 278b28-9). Aristotle says that "the celestial sphere moves and rests in some way (πως), for it continues to occupy the same place" (*Phys.* 265b1-2), so, perhaps, the fifth element "rests in some way" in the same sense that fire rests when it arrives "high up."

So if potentiality$_{\beta}$ refers, in the case of fire, to a second potentiality to move in a certain way, as well as to a potentiality$_{1}$, and a potentiality$_{2}$ that defines this movement in terms of the definite τέλος of being in a certain natural place, the potentiality to move in respect of "somewhence" and "somewhither," or potentiality$_{\gamma}$, might be a stripped down version of potentiality$_{\beta}$ consisting of a second potentiality only, i.e., a second potentiality for an open ended ἐνέργεια similar to thinking or seeing, but which is and can only be uninterrupted, unimpeded, and eternal, i.e., a potentiality for being at (unspecified) different places at different instants. If potentiality$_{\gamma}$ is as I have just described it, then there is no difference between the phases of a circular motion and no such thing as being closer to some τέλος. The revolving sphere is not potentiality$_{\beta}$ at a τέλος. It is always, rather, with full actuality exercising its potentiality for uninterrupted activity.

1.7.4 Or is the natural motion of the heavens a κίνησις?

The notion of potentiality$_\gamma$, or the potentiality to move in respect of "somewhence" and "somewhither," may be useful in describing an order of change in abstraction from the order of time, but we should be cautious about concluding that the circular motion of the heavens is an ἐνέργεια. First of all Aristotle's use of the words ἐνέργεια, ἐνεργεῖν, and κίνησις is at best inconclusive, and at worst, seems to suggest that the circular motion of the heavens is a κίνησις instead of an ἐνέργεια.

At *Metaphysics* 1050b18-22, we saw Aristotle saying that imperishable things exist ἐνεργείᾳ (b18), and that the sun and the stars and the whole heaven ἀεὶ ἐνεργεῖ (b22). But then he goes right on to apply the same words to earth and fire and their motions, which are clearly κινήσεις, by saying:

> Imperishable things are imitated by those that are involved in change, e.g. earth and fire. For these also ἀεὶ ἐνεργεῖ; for they have their movement of themselves and in themselves. But the other potencies, according to our previous discussion, are all potencies for opposites. (*Metaph.* 1050b28-32).

And we already saw in Section 1.6 that at *De anima* 417a16-7 and *Physics* 201b31 Aristotle says that κίνησις is a sort of an ἐνέργεια (ἐνέργειά τις), although incomplete. So the mere use of the words ἐνέργεια and ἐνεργεῖν is not decisive. Moreover, Aristotle calls celestial revolutions locomotions (φοραί; which are the paradigmatic sort of κίνησις[24]) or κινήσεις[25] in *Physics* 4.5 and throughout *Physics* book 8. In fact, Aristotle calls the circular motions of the heavens κινήσεις, even while he is saying that they have no limits (*Phys.* 265a27-

32), which would at least be odd if their lack of limits is intended to make them ἐνέργειαι in *contradistinction* to κινήσεις.

Further doubt is cast on the status of celestial motions as ἐνέργειαι by several instances where Aristotle seems to implicitly classify them as κινήσεις. At *Physics* 223b13 and 265b8-9, Aristotle says that circular motion is the measure of all motions. But if it is to be the measure, it must be the same sort of thing of which it is a measure. As Aristotle says, "the measure is always of the same kind (συγγενές)," so the measure of κινήσεις must be a κίνησις (*Metaph*. 1053a24-30). In addition, we saw that actions which are ends in themselves, or in which an end is present are ἐνέργειαι, otherwise, they are κινήσεις. But at *Physics* 265b14-6, Aristotle tells us that circular locomotion (ἡ κύκλῳ φορά) has no beginning or τέλος in itself (ἐν αὐτῇ) but is determined from elsewhere (ἐκτός). And finally, in *Nicomachean Ethics* 10.4 Aristotle distinguishes between κινήσεις and ἐνέργειαι by claiming that the parts of κινήσεις differ in kind from the whole while the parts of ἐνέργειαι do not. "Differing in kind," here, means having different termini, and, in the case of parts of κινήσεις, the difference amounts to a formal incompleteness. The example he gives for a κίνησις is walking a racecourse, where "the 'somewhence' and the 'somewhither' are not the same in the whole course and in a part of it, nor in one part and in another" (*EN* 1174a32-4). Seeing, and feeling pleasure, on the other hand, are "at any moment complete." Now a celestial revolution is "at any moment complete," because at every moment a celestial revolution has been completed, but one cannot divide up a celestial revolution and still get complete revolutions. So celestial revolution

fails this test of being an ἐνέργεια, and if Aristotle's stroll on the racecourse is a δίαυλος, or a walk around the post and back, then it is directly analogous to a celestial revolution.

The fact that the parts of celestial motions must "differ in kind" from the whole can also be seen from the following: We saw that celestial motion is τέλειος (*Phys*. 264b28, 265a17, *Cael*. 284a7), which is consistent with its being an ἐνέργεια instead of a κίνησις (*Metaph*. 1048b22), and we saw that it meets the test that "φ-ing" is compatible with "has φ-ed" (*Metaph*. 1048b23-36) because at every point a star "has completed" a circuit. But a star does not satisfy this condition just by being at (unspecified) different places at different instants. It does this by satisfying the much stronger condition of, at every instant, "having traversed" a circuit that terminates at the point that it happens to occupy on the circumference, and it does this by having occupied a specific sequence of positions along the circumference of the circle. So while the circular motion of the heavens does not move toward a normative completion of its nature, it is wrong to think that it has no developmental structure or order. At any rate, if circular motion is the measure of other motions (*Phys*. 223b12-21, 265a13, b8-9), it must have *some* internal structure to provide a basis for marking off units. This structure, of course, is the circuit, i.e., the procession of the celestial body from an arbitrarily chosen point, around the circumference, and back to the same point. By contrast, pleasure or seeing has no such internal development.

Finally, why settle for implicit classifications when an explicit classification is at hand? In *Nicomachean Ethics* 10.3, Aristotle distinguishes

between κινήσεις and ἐνέργειαι by pointing out that quickness and slowness are proper (οἰκεῖον) to κινήσεις, even if they are not καθ' αὑτό, *as in κινήσεις of the cosmos*, whose quickness and slowness is relative to another (πρὸς ἄλλο; *EN* 1173a32-4). The κινήσεις of the cosmos are clearly adduced, here, as an example of κινήσεις in contradistinction to ἐνέργειαι.

1.7.5 The teleology of celestial motion

And the issue of the (lack of) teleology of celestial motion is not as clear cut as we have made it out to be either. Recall that, in Aristotle's view, circular locomotion (ἡ κύκλῳ φορά) has "no beginning or limit or middle *simpliciter* (ἁπλῶς)" (*Cael*. 288a22-4). This *"simpliciter"* seems to leave the door open for some qualified sense in which celestial motion is teleological, and indeed Aristotle appears to make at least three unrelated attempts to introduce teleology into celestial motion. At *De caelo* 277a23-6, he says "Even in circular movement there is a sort of opposition between the ends of the diameter, though the movement as a whole has no contrary: so that here too the movement has in a sense an opposed and finite τέλος." What this means, I believe, is that there is a potential contrariety between points on a circle, insofar as, if an object moving along a circumference were to start or stop, the chord connecting the starting and stopping points would have contrary termini. Then there is an obscure passage at *Physics* 265b2-4, where Aristotle suggests that in the case of a rotating sphere, "the center is alike a beginning, middle, and τέλος of the magnitude traversed." Since this point is not a point on the circumference, "there is no point at which

that which is in process of locomotion can be in a state of rest as having traversed its course, because in its locomotion it is proceeding always about a central point and not to an extreme point" (*Phys*. 265b4-7). Therefore the revolving sphere "remains still, and the whole is in a sense always at rest and moves continually."

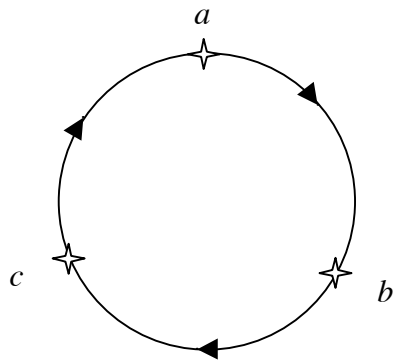
Finally, there are passages that make the much more reasonable suggestion that while there is no starting point or finishing point *simpliciter* in eternal circular motion, *every* point is a starting point and a finishing point *in a way*. Aristotle says that "any one point as much as any other is alike a beginning, middle, and a τέλος, so that we can say of certain things both that they are always and never in their beginning and in their end (ἐν τέλει)" (*Phys*. 265a32-b2). The point seems to be that the potentiality for movement in respect of "somewhence" and "somewhither" is a potentiality for returning to a place which is the same in one sense but not the same in another. The sense in which the place is the same is given in the following series of passages: "That which is in motion from A will in virtue of the same impulse (πρόθεσις) be simultaneously in motion to A (since it is in motion to the point at which it will finally arrive)" (*Phys*. 264b10-12). "Circular motion is motion of a thing from its place to its place, whereas rectilinear motion is motion from its place to another place" (*Phys*. 264b18-9). "In these [rectilinear] motions the beginning and the limit do not coincide, whereas in motion over a circle they do coincide, and so this is the only perfect motion" (*Phys*. 264b27-8). "The body whose path is the circle starts (ἤρξατο) from the same place at which it finishes (τελευτᾷ)" (*Cael*. 279b2-3). On the other hand, the sense in which the potentiality for movement in respect of

"somewhence" and "somewhither" is a potentiality for returning to a place which is *different* turns on the fact that the termini of circular motion are always ἐν ἄλλῳ καὶ ἄλλῳ (*Phys*. 264b19-27). The point seems to be that celestial revolutions always return to the same place, but there are many of these "same places" to which they return, and each of these "same places" is at once the end of a previous circuit, and the beginning of a succeeding one.

1.7.6 The order of celestial motion

1.7.6.1 Asymmetry

This, I believe, gives us an opening to claim that a teleological potentiality$_{\beta}$ applies to celestial motion, and I intend, now, to offer an alternative to the option of holding that celestial motion is ordered by something like potentiality$_{\gamma}$. In Section 1.7.4, I argued that the circular movement of the heavens, though it does not move toward a normative completion of its nature, nevertheless has a developmental structure or order, and now we can say that it is directed at a τέλος of sorts. Consider the following diagram of a star that is fixed upon the celestial sphere, and that is revolving clockwise from position *a* to position *b*, then from position *b* to position *c*, then on to position *a*:



According to the analysis of the previous section, at any instant when the star is at position *a*, it is at the goal of a previous circuit and at the beginning of a succeeding one. Since it has just completed a circuit, it is "at its goal" in the sense that it has just realized a potentiality$_{\beta}$ to reach position *a* by means of passing through positions *b* and *c* in sequence. But now it has the potentiality$_{2}$ to reach position *a again* by means of passing through positions *b* and *c* in sequence *again*. And it is clear that this "sequence" need not be a temporal one, since it is determined, at least within a given circuit and assuming that the direction of revolution is always clockwise, by the proximity to a goal.

The fact that the star starts out at position *a* and returns to position *a* does not, in itself, imply a reversibility that would destroy the asymmetry of the *x* is potentially$_{\beta}$ *y* relation. First of all, talking about a *return* to position *a* implies the existence of numerically different but type-identical circuits of a circular motion. In order to distinguish types from tokens, let us say that the upper case letters A, B, and C denote phase types of a circular motion, that the lower case letters *a*, *b*, and *c* denote the positions which the star is in in the corresponding

phase types, and that subscripts denote numerically different tokens of a given phase type. Thus, the sequence A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, C_3 … will be a sequence of phase tokens that represents successive revolutions of the star. Now as long as the star revolves in a clockwise direction, for instance, it cannot reach position *a* again without passing through positions *b* and *c* again in sequence, and its phases will be in this sequence. When the star is in phase A_2, it has just realized the potentiality$_\beta$ it had to reach phase A_2 when it was at phase A_1, and now it has a new potentiality$_\beta$ to reach phase A_3. What becomes of the goal of reaching phase A_2 when the star moves from phase A_1 to B_1? Nothing. It is merely joined by the goal of reaching phase B_2, so that when the star is in phase B_1, it has the goals of reaching *both* phases A_2 and B_2. And it cannot realize either of these goals without first passing through phase C_1. What this means is that the sequence of the phases C_1 and A_2 falls within *two* teleological motions, one from phase A_1 to phase A_2, and one from phase B_1 to phase B_2, so that, in a sense, the order of the phases C_1 and A_2 is *overdetermined* by the relation *x* is potentially$_\beta$ *y*. The relation *x* is potentially$_\beta$ *y* holds between phases C_1 and A_2 because they fall within teleological motions from phase A_1 to phase A_2, phase B_1 to phase B_2, *and* phase C_1 to phase C_2. In fact, since the positions along the circumference are densely ordered, the order of any two phases within a single circuit is *infinitely* overdetermined by the relation *x* is potentially$_\beta$ *y*.

1.7.6.2 Permanence of the asymmetry of celestial motion

1.7.6.2.1 Why there can be no direction reversals in circular celestial motion

In the foregoing account, the asymmetry of the relation *x* is potentially$_{\beta}$ *y* obviously depends on circular celestial motion being either consistently clockwise or counter-clockwise. The star, for instance, cannot go from position *a* to position *b*, and then turn around and go back to position *a* without first passing through position *c*. Although there are some inconsistencies in Aristotle's account, I believe that, in the final analysis, Aristotle does not allow this to happen. The difficulty is with *De caelo* 1.4 (271a19-33), which argues that clockwise and counter-clockwise motions around the same circumference are *not* contrary because contrary locomotions (φοραί) must have contrary termini, and both motions have the same terminus, viz., *a*, as both a beginning and an end. This conclusion, which is repeated at *De caelo* 277a23, and 286a3, would seem to imply that either counter-clockwise and clockwise motion could be natural to any particular rotating sphere, and if this is the case, there seems to be no reason why a celestial sphere should rotate in only one direction. But Aristotle appears to contradict himself at *Physics* 262a8-12, where he says that a (clockwise) κίνησις along a circumference from position *a* to and through position *b* to position *a* is contrary (ἐναντία) to a (counter-clockwise) κίνησις from position *a* to and through position *c* to position *a* because "even if they are continuous and there is no reversal (ἀνάκαμψις) they arrest each other, because contraries annihilate or obstruct one another." At *Physics* 264b13-7, Aristotle makes the potentially useful distinction that clockwise and counter-clockwise motions are opposite

(ἀντικειμένη) but not contrary (ἐναντία), but he fails to make any further use of it.

The key to this puzzle, I believe, is the fact that if the star goes from position *a* to position *b*, and goes back to position *a* without first passing through position *c*, it must perform a direction reversal or an ἀνάκαμψις. Aristotle tells us at *Physics* 262a12-7 that, if there is an ἀνάκαμψις, then a motion cannot be in a circle (κύκλῳ φέρεσθαι), but merely along the path of a circle (κύκλον φέρεσθαι). The difference between κύκλῳ φέρεσθαι and κύκλον φέρεσθαι, respectively, is the distinction between genuinely circular motion, and what is essentially rectilinear motion. In *De caelo* 1.4, any motion along an arc segment is not only treated as a non-circular motion, but it is even assimilated to rectilinear motion since "we invariably regard the distance between two points as the length of the straight line which joins them" (271a12). Therefore, if a star moves from position *a* to position *b*, and then turns around at *b* and goes back to position *a* without first passing through position *c*, it will have performed, in essence, two *contrary* rectilinear motions: one from *a* to *b*, and one from *b* to *a*. And if this is the case, the star will have come to a stand at *b* (cf. *Phys*. 262b22-263a3), and the motion will be discontinuous (cf. *Phys*. 264b23-6). An ἀνάκαμψις, then, converts a circular motion into a rectilinear motion, and the result is a loss of continuity. But circular motion (ἡ κύκλῳ κίνησις) is infinite, single, *and continuous* (*Phys*. 261b27-8, 265a27-8), and it is the only motion that is [infinitely] continuous (*Phys*. 265a7-9). So in order for circular motion to be circular motion, there can be no ἀνάκαμψις.

If it is an essential feature of the circular motion of the heavens to be continuous, then it cannot change direction. It never has changed direction, nor could it ever in the future. In fact, according to *De caelo* 1.10 and 1.12, it is a general principle that if something has always been a certain way, it can never fail to be that same way in the future, because if it had the potentiality to be other than it is for an infinite stretch of past time, it would have already manifested this potentiality at some point (this is often called the "principle of plenitude," see *Cael*. 279b21-3, 281b18-24).

1.7.6.2.2 Why have the heavens always revolved in one direction instead of always revolving in the other?

Now this tells us why a celestial sphere cannot start spinning counter-clockwise if it has always spun clockwise, but it does not answer the question of why it was always spinning clockwise in the first place. Aristotle's answer, which he gives in *De caelo* 2.5, is that one direction is somehow better than the other. This chapter, I believe, is worth quoting in its entirety:

> Now there are two ways of moving along a circle, from *a* to *b* or from *a* to *c*, and we have already explained that these movements are not contrary to one another. But nothing which concerns the eternal can be a matter of chance or spontaneity, and the heaven and its circular motion are eternal. We must therefore ask why this motion takes one direction and not the other. Either this is itself an ultimate fact or there is an ultimate fact behind it. It may seem evidence of excessive folly or excessive zeal to try to provide an explanation of some things, or of everything, admitting no exception. The criticism, however, is not always just: one should first consider what reason there is for speaking, and also what kind of certainty is looked for, whether human merely or of a more cogent kind. When any one shall succeed in finding proofs of greater precision, gratitude will be due to him for the discovery, but at present we must be content with a

> probable solution. IF nature always follows the best course possible, and, just as upward movement is the superior form of rectilinear movement, since the upper region is more divine than the lower, so forward movement is superior to backward, THEN front and back exhibits, like right and left, as we said before and as the difficulty just stated itself suggests, the distinction of prior and posterior, which provides a reason and so solves our difficulty.[26] Supposing that nature is ordered in the best way possible, this may stand as the reason of the fact mentioned. For it is best to move with a movement simple and unceasing, and, further, in the superior of two possible directions." (*Cael.* 287b22-288a12)

What Aristotle is doing, here, I believe, is restoring the normative aspect to the order of circular celestial motion. Recall that sublunary substances move toward the normative goal of completing or perfecting their nature. The heavenly spheres do not do this, because they are already perfect. But the heavenly spheres move nonetheless, and the puzzle is, if every spot on the circumference is as good as the other, why should a sphere move at all, or if it moves, why should it move in one direction rather than another? The answer is that what the heavenly spheres do to manifest their perfection is more complex than just being at different places at every instant, and there is a positive value to this ordered complexity, i.e., the fact that the celestial spheres move in precisely the ordered way that they do allows their motion to be the measure of all other motions and enables them to supply the continuity of time. Indeed, the fact that the heavens rotate in one direction rather than another enables Aristotle to claim that there is a permanent orientation to the universe in terms of right, left, front and back (*Cael.* 2.2). If celestial motion has such a normative order, then it would appear that a teleological potentiality$_{\beta}$ applies to it after all, with the result that the order of

celestial change, as well as the order of sublunary change, can be specified in abstraction from the order of time.

2.0 THE RELATIONSHIP BETWEEN THE ORDERS OF PLACE, CHANGE AND TIME

2.1 Aristotle's derivation of the kinetic order from the spatial order

I have argued that the order of the prior and posterior in change is determined by the goal-directed character of natural substances, combined with either certain aspects of the substance's form, or aspects of the medium in which the change takes place. The discussion of the prior and posterior in change in *Physics* book 4 chapter 11 focuses on an example of the last sort of case, i.e., locomotion. In locomotion, while the destination may be determined by the intentions or nature of the substance that is changing, the order of the stages of the change is determined by the structure of the spatial magnitude traversed. In fact, in the case of locomotion, the order of the change is precisely determined by the "following" relation, which appears to be a one-one mapping from "the prior and posterior in place" onto "the prior and posterior in change":

> But what is moved is moved from something to something, and all magnitude is continuous. Therefore the change follows (ἀκολουθεῖ) the magnitude. … The distinction of "prior" and "posterior" holds primarily, then, in place; and there in virtue of relative position. Since then "prior" and "posterior" hold in magnitude, they must hold also in motion, these corresponding to those. (*Phys*. 219a10-9)

At *Metaphysics* 1018b9-29, the prior and posterior in place is defined relative to "some place which is either naturally determined as a starting point,

e.g., the middle or the end, or simply happens to function in this way." The obvious relativity or conventionality of this "prior and posterior" has troubled some commentators. Hussey, for instance, claims that the prior and posterior in place can only be specified relative to some motion, and, therefore, can only be derived by arguing back from the prior and posterior in change, which is, by assumption, supposed to be derived from the prior and posterior in place.[27] But this circularity is only apparent, because, for Aristotle, *both* the prior and posterior in place and the prior and posterior in change are derived from substance. Each substance, at each phase of a motion, has a prior and posterior in place *for it* that is determined by its own goal and innate impulse toward that goal. Thus, while there is no single prior and posterior in place that exists for all substances, it is still true to say of each substance, at each phase of any of its changes, that a prior and posterior in change exists for it. Thus, the prior and posterior in place for Zeno's runner, for instance, is simply the ordered sequence of positions along the racecourse that is determined by his intention to run the race. His prior and posterior in change, in turn, is the sequence of the run's phases, viz., runner-at-position-n (n = 1, 2, 3, …), each representing the incomplete actuality of the potentiality$_{\beta}$ of the runner to finish the race. Thus, the prior and posterior in place need not be intrinsic to the place in advance of any change. Since places are defined relative to substances, and substances are what they are by having a nature, that is, an innate principle of change and rest, then both the prior and posterior in place and the prior and posterior in change can be derived non-circularly from substance as a common source. Thus, the sense in which a place

is prior and posterior is relativized to substances, but this in no way results in a vicious circularity in the derivation of the prior and posterior in change from the prior and posterior in place.

Aristotle not only relativizes time and space to substances; he relativizes the physical laws that govern time and space to them as well. For Aristotle, the world's intelligibility is not located in a four-dimensional space-time and the mechanical laws that govern it, but rather, in the specific natures of all of the individual substances that happen to exist in it. It has been pointed out that the loss of abstract generality that this position entails, proved to be a block, for Aristotle (and many who followed him, including Galileo), in hitting upon the concept of inertia, since inertia, as we conceive it locates kinetic efficacy in common properties of substances rather than in the substances themselves. Plato, in fact, is closer to the modern view in his search for universal common elements and principles (e.g., not only the Forms, but also principles like γένεσις, as described in the *Timaeus*, which Aristotle seems to deny when he says that "there is no such thing as motion over and above the things" at *Physics* 200b32-3), and, no doubt, there is an element of anti-Platonism at the root of Aristotle's approach.

2.2 Aristotle's derivation of the temporal order from the kinetic order

The passage just quoted continues as follows, claiming that a "following" relation precisely determines the order of time as well, by mapping "the prior and posterior in change" one-one and onto the prior and posterior in time:

> The distinction of "prior" and "posterior" holds primarily, then, in place; and there in virtue of relative position. Since then "prior" and "posterior" hold in magnitude, they must hold also in motion, these corresponding to those. But also in time the distinction of "prior" and "posterior" must hold, for time and movement always correspond with each other. (*Phys*. 219a14-9)

A few lines later, Aristotle reiterates, summarizing the relationships between the orders of place, change, and time: "Change, as was said, follows magnitude, and time, as we maintain, follows change" (*Phys*. 219b15-6). In these passages, Aristotle seems to envisage three inter-related orders or relational systems — the order of place, the order of change, and the order of time —, each consisting of a domain and a relation. Each order has the same relation, i.e., *x* is prior to *y* (or equivalently, *y* is posterior to *x*), but a different domain. In the order of place, *x* is prior to *y* orders places, in the order of change, it orders kinetic phases, and in the order of time it orders "nows." Moreover, *x* is prior to *y* holds of places primarily, and members of the other domains secondarily, or, to be exact, the holding of *x* is prior to *y* of nows *depends* on its holding of kinetic phases, and the holding of the same relation of kinetic phases *depends* on its holding of places.

The holding of *x* is prior to *y* of a run's phases, for instance, depends on its holding of positions along a racecourse. One can see why Aristotle might think this, if place specifications are embedded in the definitions of kinetic phases, since, in his view, for *x* to have *y* embedded in its definition is for *x* to be dependent on *y* for its existence. In the sense of definitional dependence, then, one can say that just as the existence of the cultured man depends on the existence of the man, so the existence of the state of affairs being-at-place-*x* depends on the

existence of place-*x*. And since the existence of the kinetic relata related by *x* is prior to *y* depends on the existence of spatial relata that are related by the same relation, the holding of the relation *x* is prior to *y* between kinetic relata depends on its holding between spatial relata.[28]

It is less clear why Aristotle might think that the holding of *x* is prior to *y* for times depends on its holding of kinetic phases. It seems to me, however, that a clue might be found in *Physics* 219b25 and 28, where Aristotle says that the now exists "insofar as the prior and posterior [in change] is countable." "Insofar as" translates the Greek relative adverb ᾗ, and may be glossed as "in such measure or degree as," or "to such extent that." But it seems equally possible to gloss "*x* exists insofar as *y* is F" as "the existence of *x* depends on *y* being F." If we take the phrase "the prior and posterior in change" to denote the class of kinetic phases that are ordered by the relation *x* is prior to *y,* and "the now" to denote the class of instants ordered by the same relation, *Physics* 219b25 and 28 could be saying that each now is such that its existence depends on some kinetic phase being countable, or equivalently, on the existence of some countable kinetic phase. So just as the existence of the state of affairs "being-in-the-Agora" depends on the existence of "the Agora," so the existence of the now at which Coriscus is in the Agora depends on the countability of the state of affairs "being-in-the-Agora" (or the existence of the countable state of affairs "being-in-the-Agora").[29] And since the existence of the temporal relata related by *x* is prior to *y* depends on the existence of kinetic relata that are related by the same relation, the

holding of the relation *x* is prior to *y* between temporal relata depends on its holding between kinetic relata.

That the existence of the now depends on the countability of kinetic phases should come as no surprise, since Aristotle defines time as "the number of motion in respect of 'prior' and 'posterior,'" since Aristotle thinks of number as something essentially countable (*Phys*. 206a3-5, *Metaph*. 1020a7-32; 1057a2-4), and since it is clear that the focus is on counting nows in *Physics* 4.11, where he defines time. What Aristotle thinks it means to be countable is clear enough from *Metaphysics* 10, which discusses the "one" and its relation to number and counting. A thing's countability would appear to be its unity *qua* an instance of some sortal concept, which allows it to be taken as a member of a plurality of units. A horse, for instance, is countable *qua* a member of a plurality of horses because of the unity of its substantial form. Likewise, a kinetic phase is countable *qua* a member of a plurality of kinetic phases because of its unity as a state of affairs. But it is still somewhat mysterious how, precisely, each kinetic phase that is prior and posterior in change gets mapped to each now that is prior and posterior in time.[30]

Hussey offers a possible answer, in the form of an interpretative gloss of *Physics* 219b25 and 28: "the now *is* the prior and posterior [in change] considered as countable" (my italics). The difference between my more literal reading of this sentence and Hussey's gloss amounts to the difference between the following two claims: (a) every now depends for its existence on the countability of some kinetic phase (my translation) and (b) every kinetic phase is such that it is a now if

and only if it is countable (Hussey's gloss). Now if "b," or Hussey's gloss, is Aristotle's meaning, which seems possible but not mandatory, then the domain of nows and the domain of instantaneous kinetic phases are what Aristotle would call "identical in substratum but different in definition," since "nows," on this view, just *are* kinetic phases under a different description. And if this is the case, then the kinetic order *is* the temporal order, because the spatial, temporal and kinetic orders, *ex hypothesi*, only differ in their domains.

2.2.1 A modern objection

The idea that the order of time should reduce conceptually to the order of change can also be found in some more recent theories of the temporal order of a reductionist stripe. And, in fact, the position of some of these theories, that the relation *x* is later than *y* reduces to the relation *x* is a more entropic state than *y* bears a striking similarity to my interpretation of Aristotle on Hussey's gloss of *Physics* 219b25 and 28, since, on this view, the temporal order *is* the kinetic order, and the relation *x* is prior to *y*, when applied to kinetic relata, essentially reduces to the relation *x* is potentially$_{\beta}$ *y*.

There is a common objection to this sort of reductionism, however, which Lawrence Sklar puts in the form of an analogy: Suppose we sought to define our concepts of left and right in terms of an apparently lawlike asymmetry in the spatial orientation of certain physical systems, like the preferential axial direction of electron emissions from spinning nuclei. Then suppose that these processes should miraculously reverse direction, violating what we had previously taken to

be a natural law. Sklar then asks, "Would we then say that the clockwise direction had become the counter-clockwise? That right-handed gloves had suddenly become left-handed?" His reply:

> Nothing of the sort. We would indeed be astonished and look desperately for some explanation of this mirror reversal of a law. But we would, I believe, still take it that we could recognize left- and right-handed objects as before, teach the meanings of orientation terms by ostension, as before, etc.[31]

Suppose then, that the tendency of entropic processes to evolve from more to less ordered states reverses, and also that men begin developing into boys. Would we say, as a result of this, that the time order has reversed? Most of us would not, but since the modern time order reductionist *defines* the time order in terms of an entropy gradient, he would have to say that it has, as would Aristotle, if we assume that Hussey's gloss of *Physics* 219b25 and 28 is correct, and he thinks that the temporal and kinetic orders are the same. And, indeed, this would also be the case if we rejected Hussey's gloss and assumed that Aristotle kept the temporal and kinetic orders distinct, since a one-one mapping from kinetic phases onto nows would tie the elements of the two domains together as well as the identity relation would.

Does this result, then, render the very project of reducing the temporal order to some non-temporal order *prima facie* absurd? Not automatically. Inter-theoretic reduction is perfectly respectable when we identify light waves with electromagnetic waves, and heat with mean molecular kinetic energy, or when we claim that the essence of water is expressed by the formula "H_20." Reduction in each of these cases takes phenomena that were originally thought to be primitive,

gives them a more articulated conceptual structure, and places them within a richer theoretical context. And by being able to claim that what we had known all along as heat was actually identical to mean molecular kinetic energy, or that "H_20" expresses the essence of what we had known all along as water, we are able to inoculate ourselves against counter-factual objections in which these identities or essences do not hold, like Sklar's counter-example to the anisotropy of space. This is because, as Kripke has argued,[32] if the relation of identity holds, then it holds necessarily, and likewise, if the essence of water is expressed by the formula "H_20," then it has this essence in every possible world in which it exists.

One might suppose, then, that any project of reducing the temporal order to some non-temporal order succeeds to the degree to which one can assimilate this reduction to one of the more respectable cases just mentioned. And this, in turn, depends on whether we can plausibly say that what we had known all along as the temporal order was actually identical to, or had its essence expressed by some non-temporal order. The plausibility of saying, for instance, that what we have known all along as heat (i.e., the property of physical objects able to cause increases in temperature, etc.) was actually identical to the motion of molecules, is helped by the fact that heat is never found in the absence of molecular motion and vice versa. The case, however, for identifying the relation *x* is later than *y* with *x* is a more entropic state than *y* is *not* helped by the fact that the relationship between the order of entropy and the order of time is statistical and not lawlike. Although we rarely see an isolated system evolve from a less to a more ordered state, statistical mechanics does not rule this out, and even holds that it is highly

probable that there are quite large regions of space-time with their levels of entropy trending in opposite directions. It seems unsatisfactory, in the face of this, to try to save the data by claiming that what we have traditionally referred to as the "past" and "future" is not globally consistent.

While a circumstance like this would appear to refute the claim that the temporal order reduces to the order of entropic processes, however, I do not think that Aristotle's theory would meet with the same fate. To see this, consider what would need to be true if Aristotle's theory is to be falsified. Stated generally, we would need a case where *a* and *b* are particular phases of a change of a natural substance, and where *b* is potentially$_{\beta}$ *a*, and *a* is earlier than *b*.[33] Let us first try to imagine the limiting case, where *all* processes (including recognitional processes) reversed their temporal order, that is, suppose that the converse of *x* is potentially$_{\beta}$ *y* held for *all* of the phases of *every* process in the universe while *x* is earlier than *y* continues to hold. It seems to me that this state of affairs would be epistemically indistinguishable from the state of affairs in which the original relation held. This is because, if the relation *x* is potentially$_{\beta}$ *y* is the basis for every kinetic ordering, as I have argued that it is in Aristotle's system, it will also govern all of the perceived asymmetries that mark out the future from the past. In this circumstance, our knowledge of the future would become as certain and as copious as our current knowledge of the past and our knowledge of the past would become as uncertain and as sparse as our current knowledge of the future. This is because the contingency of potential events is the basis for Aristotle's argument for an open future and a necessary past in *De interpretatione* 9.[34] In this scenario,

the causal and explanatory orders would reverse as well, since in Aristotle's view, the fact that *a* was potentially$_{\beta}$ *b* is the cause and explanation of why what was once *a* is now *b*.

Finally, if the epistemic and causal asymmetries were reversed, it is clear that we would care more about what happened in the past than what will happen in the future, and would act for the sake of the past instead of the future. In such a circumstance, trying to bring about favorable future outcomes would seem just as irrational as trying to bring about favorable past outcomes under a normal circumstance because the future would be just as fixed and epistemically accessible as the past is in the normal circumstance. What I infer from all of this is that the limiting case where all processes reverse their temporal order differs from our current situation only insofar as the names "future" and "past" have switched their references.

But what about cases other than the limiting case just mentioned, where the converse of the relation *x* is potentially$_{\beta}$ *y* held for only some substances while the relation *x* is earlier than *y* continued to hold universally? Suppose, for instance, that men begin developing into boys. But in such a case, since it is in the essential nature of boys to develop into men and not *vice versa*, then we are no longer talking about boys and men. So men are not developing into boys after all. If the essence of a thing is defined in terms of characteristic developmental and behavioral potentialities, then it will exhibit these potentialities in every possible world in which it exists. If every thing in the world has such an essence, then nothing could feature in a counter-example to the claim that the relation *x* is

potentially$_{\beta}$ *y* either expresses the essence of the relation *x* is earlier than *y* or is to be identified with it.

Aristotle's theory is immune, then, from counter-examples of the sort suggested by Sklar, while the thermodynamic theory of the temporal order is not, but this immunity comes at a price. The price is that, when compared with the thermodynamic theory of the temporal order, Aristotle's theory ends up looking decidedly unempirical. It *was* an empirical discovery that isolated systems tend to evolve from more to less ordered states. It was also an empirical discovery that boys tend to develop into men. But it cannot have been an empirical discovery that it is part of the essence of boys to do this. Since Aristotle *defines* the essence of boys in terms of their position in a normative developmental order, it should not surprise us that they cannot possibly appear in that order anywhere else but in their proper position. By contrast, there is nothing in the definition of entropy that requires less ordered states to follow more ordered states, and if this did not occur, we would not be tempted to say that entropy is no longer entropy. Aristotle knows *a priori* that men cannot develop into boys because he has essentially stipulated it, and this buys him immunity from counter examples, but it also means that his theory is not testable, falsifiable, or refutable. And if we accept Popper's criterion that a scientific theory is one that makes risky predictions, then I suppose we would have to say that Aristotle's theory is unscientific.

On the positive, side, however, Aristotle's theory does secure a benefit that we typically look for in an inter-theoretic reduction. Paradigm cases of inter-theoretic reduction, like the identification of light waves with electromagnetic

waves, and heat with mean molecular kinetic energy, take phenomena that were originally thought to be primitive, give them a more articulated conceptual structure, and place them within a richer theoretical context. By taking the temporal order and reducing it to another type of order for which he already has a theoretical account, Aristotle takes something that would otherwise be thought of as an irreducibly basic and primitive fact of nature, and converts it into something that is explicable in terms of his theories of change, act, and potency. And this, in turn, gives him the resources to give a unified explanation of a host of other temporal asymmetries, like the epistemic difference between the past and the future. In sum, Aristotle's reduction of the order of time to the order of change achieves his objectives admirably, just not in a way that we would call "scientific."

3.0 THE KINETIC CONTINUUM

Waterlow correctly notes that the account of the kinetic order in *Physics* book 3 does not require the prior and posterior in change to be a dense order, i.e., an order such that between every two distinct phases of a change there is another phase.[35] Nonetheless, Aristotle claims that the prior and posterior in each of the four kinds of genuine change, i.e., change of quality, place, or size, and generation and destruction, have this property.

Denseness, here, is thought of in terms of infinite divisibility. A motion or a magnitude is infinitely divisible insofar as the (potential) points at which it is divisible are densely ordered, i.e., insofar as between any two points on a body

and any two phases of a change, there is another (potential) point or phase at which the body or motion could be divided. Since Aristotle insists that both motions and magnitudes are infinitely divisible, he implies that the prior and posterior in place and the prior and posterior in change are dense orders (*Physics* 6.6 237a17-b3; b9-21).

It is clear from Aristotle's discussions of these four kinds of change (change of quality, change of place, change of size, and generation and destruction), that each of them ultimately derives its infinite divisibility, and therefore, the dense ordering of its phases, from the infinite divisibility of magnitude. In changes between contradictory states which admit no *tertium quid*, the infinite divisibility of the change is borrowed from the infinite divisibility of magnitude.[36] For instance, if a body is coming into existence (a change which admits no *tertium quid*), Aristotle claims that it must come into existence piecemeal, and piecemeal according to an infinitely divisible process of addition (*Physics* 6.4 234b10-20, 240a19-29, 237b10). In changes between contrary states such as colors and tastes, which admit *tertia quae*, but not infinitely many of them (*De sensu* 445b21-9; 446a16-20), infinite divisibility appears to be derived indirectly from the infinite possibilities for dividing up the proportions of earth, air, fire, and water in the changing thing.[37] Finally, in cases of change of place and change of size, since the media in which the changes take place are infinitely divisible, Aristotle argues that the law of non-contradiction requires an infinite number of *tertia quae* between any two termini (*Phys*. 237a17-28).

The dense ordering of the phases in a change of place also follows from the fact that change "follows" magnitude, i.e., that "the prior and posterior in place" may be mapped onto "the prior and posterior in change." Aristotle claims that we may also infer, based on this mapping, from the continuity of magnitude to the continuity of change (*Phys*. 219a12-3). Normally, we think of denseness as something distinct from continuity, but Aristotle conceives of continuity in such a way as to make it equivalent to infinite divisibility, and, therefore to denseness. At *Physics* 5.3 226b21-227a15, Aristotle defines continuous magnitudes as things having parts whose description could only be satisfied by other continuous magnitudes, i.e., they must have distinct boundaries that coincide with boundaries of other parts that succeed and precede them. But if a continuous magnitude is just a thing that has parts that are also continuous magnitudes, albeit situated in a certain way, this is just to say that being continuous is being infinitely divisible. Hence, continuity, for Aristotle, is an irreducibly geometrical concept. On Aristotle's view, a continuous magnitude is an entity that *has* points but is not *composed* of them. Moreover, the relationship between a continuum and a point is not one of set membership (*Phys*. 239b5-9, 30-33, *GC* 316a25-34, b14-16, 320b14-16). Rather, it is a relation of substrate to property or accident (*Phys*. 220a22). A punctual state of affairs is either a bound or a potential bound of a kinetic continuum, and, as such, it is a property of that continuum, rather than a constitutive part of it. Thus, at *Physics* 219a19-21, Aristotle says that "The prior and posterior in change is, in respect of its substrate[38] change; but its being is different and is not change." "Change," here, refers to the kinetic continuum, and

the prior and posterior in change, i.e., the punctual phases of this continuum that are prior and posterior, are accidents of it.

But given that the prior and posterior in change is a dense linear order, and indeed, that a distance between any two elements in the order is to be thought of on the analogy of an infinitely divisible geometrical line, how do we interpret this distance as anything other than a stretch of time? It is clear that this distance cannot be a spatial distance. Aristotle implies that while the order of prior and posterior is primarily spatial, it is transposed into the kinetic order by becoming the order not simply of places, but of *states of affairs containing places*. So what are ordered in the kinetic sequence are categorically different from what are ordered in the spatial sequence, i.e., states of affairs in the first sequence and places in the second, and because of this, the distance between things that are ordered by the prior and posterior in change cannot be simply a spatial distance. This becomes quite clear when we consider that kinetic distance must also be a distance between qualities and quantities, *as well as* between places (*Phys*. 223a29-b1). So what else can the distance between two states of affairs be but a stretch of time?

3.1 A modal interpretation of the kinetic continuum

But perhaps the concept of potentiality$_\beta$ that I used to order the phases of a change can also be used to interpret the status of those phases. The relation *x* is potentially$_\beta$ *y* not only relates the phases of a change (i.e., things like Socrates-as-a-boy and Socrates-as-a-man), but, assuming that there is an underlying, enduring

individual (viz., Socrates), it also relates possible states of affairs involving this individual. It is natural to think of a phase of a motion as a possible state of affairs, and we often speak of possible states of affairs in the semantics of modal logic. And significantly, if we think of the phases of a change as possible states of affairs, it is unnecessary to assign them time references. With this advantage in view, I propose that with some minor adjustments, the actualist interpretation of possible worlds semantics developed by Alvin Plantinga can be used to sketch a modal interpretation of Aristotle's kinetic continuum, that is, as a densely ordered sequence of possible states of affairs.

Specifically, I propose that the kinetic continuum is a continuum of possibilities$_{\beta}$ and actualities of states of affairs, and that what are prior and posterior in motion are distinguished from what are prior and posterior in place, by being neither places *simpliciter*, nor even states of affairs *simpliciter*, but by being realized and unrealized possibilities$_{\beta}$ of occupying certain places. The realized possibilities$_{\beta}$ form the history of a change, while the ones to be realized are its "intended" trajectory, i.e., "intended" by the nature of the changing substance (including any of its conscious or unconscious goals), combined with the structure of the medium to be traversed.

3.1.1 States of affairs

Aristotle, of course, does not speak of "states of affairs" and "possible worlds," but I think that the basis for these notions is present in Aristotle, even if they are not worked out in comparable detail. Let us consider, first, states of affairs.

Plantinga takes states of affairs to be abstract entities, like universals, which, instead of either being exemplified or not, either obtain or fail to obtain. On the modern view, a state of affairs is a structured complex that consists of at least one particular, at least one universal (i.e., a property, kind, or relation), and one or more exemplification relation between the particular(s) and the universal(s). That a state of affairs is a "structured complex" means that it is more than the sum of its parts, and has an overall form or organization.

What would Aristotle make of this? At the most basic level, a state of affairs is just the way things are, and Aristotle clearly does not doubt that there is such a thing as the way things are. But what about the claim that a state of affairs is an abstract entity like a universal? If the point of saying this is to ensure that the *same* state of affairs is repeatable, then Aristotle still should not have a problem with it. Although Aristotle makes universals dependent entities, he is nonetheless a realist about them. Since he thinks that it is possible for the property "white" to be multiply exemplified, it will clearly be possible for a state of affairs like "a man being white" to obtain more than once. "A man being white," which is a perfectly general and repeatable state of affairs, is one of the examples Aristotle uses in his discussion of modality in *De interpretatione* 12. As for structure, Aristotle would surely agree that a state of affairs is a structured entity. One cannot simply think "man" and "white," he says, and have a thought with a truth value, and this, presumably, is because the truth makers of thoughts (viz., states of affairs) have structures themselves (*de Int*. 1).

3.1.2 Possible worlds and entailment of states of affairs

Now if a possible world is just a very large and complex state of affairs, or a "total" way that a world might be, I see no reason why Aristotle should not accept the concept of a possible world. On this conception, a possible world is just an aggregate of all of the compatible states of affairs of all the things that are, where some states of affairs will include particulars in motion (and, hence, will be kinetic phases) and some will include particulars at rest. Plantinga calls this a "maximally consistent"[39] possible state of affairs, and to articulate it, he makes the interesting claim that states of affairs can entail or preclude one another. A maximally consistent state of affairs, or possible world W, is one specified as follows: for every state of affairs S, W either entails S or precludes S. The type of entailment, here, is strictly logical. A possible world W entails S because S is a state of affairs that is contained in W. For example, if W is a world in which Socrates exists, then W is also a world in which someone exists, or put another way, it is impossible for Socrates to exist without a person existing.

But what about entailment of the *actuality of* possible worlds by the *actuality of* other possible worlds? I do not mean logical entailment, where one world is included within another (in which case, one of them would not be maximally consistent). What I mean is the sort of entailment involved in hypothetical necessity, where if such and such a posterior state of affairs is to be actual, then such and such prior state(s) of affairs must be actual. If every change has a goal state of affairs *c*, and if the actuality of goal state of affairs *c* presupposes the actuality of state of affairs *b*, and the actuality of state of affairs *b*

presupposes the actuality of state of affairs *a*, then the actuality of goal state of affairs *c* presupposes the actuality of state of affairs *a*.[40] Moreover, if world W_a contains state of affairs *a*, and world W_b contains state of affairs *b*, and world W_c contains state of affairs *c*, then the actuality of possible world W_c presupposes the actuality of possible world W_b, and the actuality of possible world W_b presupposes the actuality of possible world W_a, and the actuality of possible world W_c presupposes the actuality of possible world W_a. And if states of affairs *a*, *b*, and *c* include particulars in motion (and, hence, are kinetic phases) and states of affairs *d*, *e*, and *f* include particulars at rest, then the states of affairs *d*, *e*, and *f* will stand in the same relations of priority and posteriority as states of affairs *a*, *b*, and *c* respectively, if state of affairs *d* is part of possible world W_a, state of affairs *e* is part of possible world W_b, and state of affairs *f* is part of possible world W_c.

3.1.3 Objections addressed

I can see two objections arising from this, one more subtle than the other. The first objection is that I have just described a deterministic universe, whereas Aristotle rejects determinism. Note that the only kind of necessity I have invoked is *hypothetical* necessity, i.e., necessity that runs from a posterior to a prior state. Determinism would only result from necessity that ran the other way, from the prior to the posterior. As for the more subtle objection, I can see how one might be suspicious that I am covertly using the temporal concept of simultaneity when I talk about state of affairs *d* being part of possible world W_a, for instance. But a moment's reflection on the concept of a possible world and a state of affairs

shows that this is misguided. A state of affairs is a structured complex consisting of particulars, universals, and exemplification relations. A possible world is just one or more state of affairs. Consider a possible world in which the only existents are two geometrical cubes that share one of their sides. Each cube is a state of affairs consisting of six particular geometrical surfaces related to each other in a certain way and exemplifying a certain shape and size. Are we required to say that these states of affairs are simultaneous with each other? I think not, and this is true even on the assumption that geometrical objects are not Platonic. That is, even if we, as Aristotle does, take geometrical cubes to be abstractions from concrete cubes, we can still abstract them from their existence in time, as well as from any of their material characteristics. And if this is possible, I do not see why we could not abstract a fully material bronze cube from its existence in time as well, leaving its material characteristics and removing only its time reference. A state of affairs is an abstract entity; it contains no reference to time. One may wish to claim that concatenated states of affairs are simultaneous, but one is not required to do so by the mere fact that one concatenates them.

3.1.4 Plantinga's Platonism

The "actualism" in possible worlds actualism derives from the claim that only things that make up the actual world exist. This is in contrast to possible worlds possibilism, which claims that things exist other than things that make up the actual world, viz., other possible worlds that are made up of things that are of comparable metaphysical standing to things in the actual world. Plantinga thinks

that it is incoherent to suppose things can exist, otherwise than in the actual world. So in order to say that possible worlds and possible, but not actual, states of affairs exist, one must say that they exist in the actual world in some sense. The way Plantinga does this is to claim that all possible worlds exist in the actual world in the same way that Platonic forms do. But in my view, this amounts to trading one ontological extravagance for another. Instead of claiming that possible worlds exist as unobservable self-subsistent concrete entities as Lewis does, Plantinga claims that they exist as unobservable self-subsistent abstract entities. It seems that one could work out a more ontologically conservative account of how possible states of affairs and possible worlds can exist in the actual world using Aristotle's notion of actuality and potentiality. For Aristotle, possible states of affairs exist insofar as there are particulars that have an in-dwelling potentiality to enter into those states of affairs. So insofar as there are particulars in the actual world that have certain potentialities to enter into states of affairs, there will be possible worlds existing in the actual world that feature those states of affairs.

A drawback of this approach would be that possible worlds would not be necessary beings, because the existence of a possible world in a world would be dependent on the contingent existence of a particular in that world. We would be prevented from saying, for instance, that possible states of affairs involving Socrates could exist in a world where Socrates himself doesn't exist. Still, it seems that Aristotle could claim the existence of a Pickwickian Socrates, i.e., one consisting of lumps of matter fit to be endowed with the form of Socrates, in any

possible world containing matter. In any case, it seems that the most important thing is to be able to entertain possibilities concerning Socrates in worlds where he *does* exist, especially in the actual world.

NOTES TO CHAPTER 1

[1] G E L. Owen, “Aristotle on Time,” in *Motion and Time, Space and Matter*, eds. P. Machamer and R. Turnbull (Columbus: Ohio State University Press, 1976), 22-5. Corish mounts a less successful attack. He tries to make trouble for Aristotle by imagining a motion across a continuous magnitude from point A to point B. If we describe the motion as the occupation of every position on that linear magnitude, we will still need to rule out back-and-forth motion by the stipulation that every position is occupied only once, which Corish points out, is a temporal notion. However, since a back-and-forth motion would represent two numerically distinct motions for Aristotle (*Phys*. 264a14-21), as long as we are talking about defining the order of numerically distinct motions, this stipulation will not be necessary.

[2] See Julia Annas, "Aristotle, Number and Time," *Philosophical Quarterly* 25 (April 1975): 101 n. 11; Denis Corish, "Aristotle's Attempted Derivation of Temporal Order From That of Movement and Space," *Phronesis* 21 (1976): 241; Richard Sorabji, *Time, Creation And The Continuum: Theories In Antiquity And The Early Middle Ages* (Ithaca: Cornell University Press, 1983), 86; Sarah Waterlow, "Aristotle's Now," *Philosophical Quarterly* 34 (April 1984): 119 n. 22.

[3] *Leibniz: Selections*, ed. P. P. Wiener (New York: Scribner, 1951), 201.

[4] i.e., before it begins absorbing nutriment

[5] i.e., when it is absorbing nutriment

[6] "The fitness to be endowed with a nature" would seem to be just a property of matter. Yet in Aristotelian embryology it is not this simple. *GA* 2.3-4 maintains that the male contributes the formal element, which is conveyed by the sperm, and the female contributes the material element (the menses) in the formation of the embryo, and that the sperm takes an active role and the menses takes a passive role in the process of fertilization. Yet this chapter of the *GA*, as well as *Metaphysics* Book Θ chapter 7, also seem to cast the sperm in a material role insofar as the nutritive soul is said to exist potentially in it. In *GA* 2.4, Aristotle says that the female provides the body and the male provides the soul of the offspring, but, of course, the sperm is not ensouled, so the soul in the sperm must be latent or potential. Hence, I believe that the special features of sexual reproduction, which involve an agent/patient interaction between the sperm and menses, cast both the sperm and the menses in material roles, albeit in different senses.

[7] This does not mean that a motion is reducible to its stages in the sense that its goal need not be taken into account. Aristotle says in *Physics* 5.4 that the goal state supplies one of the identity conditions for a motion. It is assumed, here, rather, that each stage is taken as a stage on the way to the goal of being *y*.

[8] The way that Waterlow puts it (Sarah Waterlow, "Instants of Motion in Aristotle's *Physics* VI," *Archiv für Geschichte der Philosophie* 65 (1983): 137), is that that Aristotle's definition of change as the actuality of that which is changeable insofar as it is changeable, is not simply a potentiality to be in a particular state, but rather, it is a potentiality to be in that state that is incompletely actual and actual in the sense of an "F-wardness." In other words, the incomplete actuality has a direction or a goal. What makes an at-at ontology of motion unsuitable for Aristotle is that it can only account for a derivative sense of this directionality at an instant.

[9] In book 2 of the *Physics*, the extension of the term "substance" appears to be broader than in the central books of the *Metaphysics*, where substances, in the primary sense of the word, are restricted to biological organisms. In book 2 of the *Physics*, a substance is anything that has a nature and "a principle of motion and of rest (in respect of place, or of growth and decrease, or by way of alteration)."

[10] Sarah Waterlow, *Nature, Change and Agency in Aristotle's Physics*, (Oxford: Oxford Clarendon Press, 1982), 102.

[11] Waterlow (Waterlow, *Nature, Change and Agency*, 122-4) suggests that the concept of substance as constituted by positive substance constitutive properties, depends on the assumption that "change ... cannot continue indefinitely towards an unattainable end." Otherwise, privation would be an ineliminable component of a substance's essence. Thus, Waterlow claims, Aristotle's concept of change

underwrites his metaphysics in the categories by ensuring that substances have a positive essence.

[12] *DA* 417a16, a21, b5, b7, b10, b19, b20, b22

[13] *DA* 417a7, a9, a13, a14, a18, a29, a30, b4, b13; 418a4

[14] *DA* 417a12, a15, b1

[15] *Metaph.* 1024b8-9, 1038a6-7, 1058a1-2, 23-4.

[16] Ian Mueller, "Aristotle on Geometrical Objects," *Archiv für Geschichte der Philosophie* 52 (1970): 167.

[17] Philoponus, *In de anima* 2.5 299,27-32; Simplicius *In de anima* 2.5 121,14-19.

[18] L. A. Kosman, "Aristotle's Definition of Motion," *Phronesis* 14 (1969): 40-62. cf. Waterlow, "Instants of Motion," 112-119; Edward Hussey, *Aristotle's Physics III & IV* (New York: Oxford Clarendon Press, 1983), 58-62.

[19] The difference between speaking φυσικῶς and speaking λογικῶς, is the difference between speaking specifically and concretely and speaking generally and abstractly respectively. See *GC* 316a7 ff., *A. Pst.* chapter 22, 84a7, chapter 32, 88a19, *EE* 1217b 20, *Metaph.* 1029b13, 1030a25, 1041a25, 1069a25 for uses of λογικῶς, and *EN* 1147a 20, *Metaph.* 1066b25, 1091a15 for uses of φυσικῶς.

[20] Kosman glosses over a complication, here, and I will too, for the moment. The inception of the process of housebuilding will be instantaneous if we define it as a transition from rest, but it will take time if we define it as a transition to some partial stage of completion. See Sorabji, *Time, Creation And The Continuum*, 405 ff.

[21] In what follows, I will refer to the rotation of the celestial spheres, and the revolution of the celestial bodies fixed upon those spheres collectively as "circular motion of the heavens" or "celestial motion." Aristotle seems to do so, with the terms ἡ κύκλῳ φορά and ἡ κύκλῳ κίνησις, and he switches between talking about revolution and rotation as though it made no difference to his arguments. Strictly speaking, heavenly bodies which are fixed upon the celestial spheres do not move καθ' αὑτὸ, but, rather, move by virtue of the motion of the spheres. (*Cael.* 291b12-3)

[22] See above, Section 1.6

[23] On the assumption that celestial revolutions are ἐνέργειαι, this restriction would allow one to pick out the κινήσεις that are involved in the ἐνέργεια of revolving, where any such κίνησις has a τέλος, though the revolution itself is a τέλος. Thus, the ἐνέργεια of revolving *contains* κινήσεις.

[24] *Phys.* 265a13-15, 25, 28, b11-12

[25] *Phys*. 212b10, 14, 259b28-31, 261b28-9, 264b9, 18-9, 19-20, 23-6, 26-7, 265b1-2

[26] I have capitalized the "if" and "then" of this rather long, compound conditional to facilitate reading it.

[27] Hussey, *Aristotle's Physics III & IV*, 147.

[28] Note that the dependence claimed here is a metaphysical dependence, not an epistemological one. Although it may be true that our knowledge that "*x* is prior to *y*" holds of temporal relata depends on our knowledge that it holds between kinetic relata, this is not what is at issue.

[29] I take a state of affairs, here, to be a particular, not a universal.

[30] Cf. Corish, "Aristotle's Attempted Derivation," 244.

[31] Lawrence Sklar, "Up and Down, Left and Right, Past and Future," *Nous* 15 (1981): 113.

[32] Saul Kripke, *Naming and Necessity*, (Cambridge, MA: Harvard University Press, 1972), 3, 35-38, 47, 49-50, 100-5, 107-15, 122-34, 137-8, 140-55, 158-60.

[33] Aristotle categorically rules out such a possibility at *De caelo* 283b13-14 where he says "No potential relates to being in the past, but always to being in the present or future."

[34] In this chapter, Aristotle shows that we know so much more about the past than the future because the truth values of future-tensed claims are not yet settled, while those of past tensed claims are, and this fact depends on the status of the future as potential, and the past as necessary.

[35] Waterlow, *Nature, Change and Agency*, 125.

[36] He also borrows infinite divisibility of change from the infinite divisibility of time (e.g., *Phys*. 237b2), but, as we shall see, the infinite divisibility of time derives from the infinite divisibility of magnitude.

[37] See Sorabji, *Time, Creation And The Continuum*, 411.

[38] Or more literally, following Ross: "The prior and posterior in change is change, [or rather] that [while] being which the prior and posterior in change are prior and posterior."

[39] Alvin Plantinga, "Actualism and Possible Worlds" in *Essays in the Metaphysics of Modality* (Oxford: Oxford University Press, 2003), 107.

[40] See above, the example of the falling lump of earth in Section 1.4.2.1.

Chapter 2: Persistence

1.0 TRANSWORLD AND TRANSTEMPORAL INDIVIDUALS

The fact that Socrates is a member of two or more possible worlds, e.g., an actual world in which he drinks hemlock, and a possible world in which he does not, makes him what Plantinga calls a "transworld individual." Possible world actualists like Kripke and Plantinga argue that our pre-philosophical notion of possibility entails the concept of a transworld individual. When we entertain the possibility, for instance, that Socrates might have fled Athens on the eve of his execution, we do not imagine that a numerically distinct counterpart with a snub nose and a penchant for philosophy fled Athens. We imagine that *he, this man Socrates*, fled Athens. Indeed, claim Kripke and Plantinga, the idea that a concrete, numerically distinct individual named "Socrates" has fled to Thessaly in some possible world while this other one here is drinking the hemlock, confounds what it means to exist in a possible world. Possible worlds, states of affairs, and their contents are abstract and not concrete entities. For Socrates to exist in a possible world is just for a maximally consistent state of affairs to entail Socrates' existence, viz., in that it is impossible for the state of affairs to exist while Socrates does not.

It is clear that the modal interpretation of Aristotle's kinetic continuum I argued for in Chapter 1 envisages transworld individuals as well, because we took the states of affairs in which Socrates is a boy and Socrates is a man to be states of

affairs involving a single individual, viz., Socrates. And if we re-admit the mention of time, now, and recognize that the kinetic phases we have been treating as possible states of affairs are also, in fact, time slices arranged in a temporal order of before and after, it is also clear that my modal interpretation of Aristotle's kinetic continuum implies a theory of persistence as well, viz., one in which a persisting entity is identical across successive time slices, or is a transtemporal individual. (The purpose of resorting to a modal interpretation of the kinetic continuum in Chapter 1 was to demonstrate that the phases of a change may be abstracted from temporal references and arranged in a non-temporal order, or, alternatively, that motion and time can be conceptually disassociated. With that objective accomplished, we are now free to think of these phases as arranged in a temporal order.)

2.0 PUZZLES ABOUT IDENTITY

But what right do we have to believe in transworld and transtemporal individuals? On the face of it, they seem to violate the law of the indiscernibility of identicals. There is an abundance of reasons why Socrates-as-a-boy is *not* identical to Socrates-as-a-man. For, presumably, the latter differs from the former not only in his size but in a myriad of other qualities such as hair color, wisdom, etc. And in the light of this, one may ask, quite generally, how an enduring entity can gain and shed properties, either its size, or its shape, or even just become older, while still remaining numerically the same entity.

Scruples like these about the existence of transtemporal individuals are generally associated, in antiquity, with certain puzzles, like Epicharmus' Growing Argument. The latter challenges our assumptions about personal identity by arguing that our identities are in constant flux. It draws this conclusion from the premises that identity is a strict function of material composition, and our material compositions are constantly changing. In what follows, I will look at the Growing Argument and a similar puzzle, about change of place mentioned by Aristotle in *Physics* 4.11, with a view to assessing the adequacy of Aristotle's approach to solving them. As a point of comparison, I will also assess the solution to the Growing Argument proposed by the Stoic philosopher Chrysippus, which features a *reductio ad absurdum* of certain premises of the puzzle.

2.1 Change of place

2.1.1 The puzzle at Physics *219b18-22*

In *Physics* book 4, chapter 11, Aristotle tries to account for the paradoxical feature of the "now," viz., that in a way it is always the same, and in a way it is always different. And at *Physics* 219b18-22, he claims that the moving thing has this same feature:

> [The moving thing], with respect to its substrate, is the same, … but in definition it is different, as the sophists assume that Coriscus being in the Lyceum is different from Coriscus being in the marketplace. And this [viz. the moving thing] is different, by means of being in different places. (*Phys*. 219b18-22)

What interests me is the "assumption" that is attributed to the Sophists. Taken literally, the allusion does not make sense. Aristotle is making a point about difference and sameness in substances, yet his example is about difference and sameness in *states of affairs* (Coriscus being in the Lyceum and Coriscus being in the marketplace). Clearly, states of affairs and substances do not persist in the same way, and it is hard to imagine what a persisting substrate of a state of affairs might be. The most natural interpretation is that, in spite of what Aristotle actually says, he means to say that it is Coriscus as a moving thing, rather than states of affairs involving Coriscus, that is the same in substrate but different in definition. This is Simplicius' interpretation, who claims that the sophistical puzzle is one in which "the same" Coriscus "becomes different from himself" by changing his place:

> [The sophists] said that "the same Coriscus is sometimes in the market-place and sometimes in the Lyceum. He who is sometimes in the market-place, sometimes in the Lyceum, becomes different from himself." (*In phys*. 723,14-16)

But this is not quite satisfactory. First, since the "assumption" is attributed to the Sophists, one would think that it must have been the subject of a puzzle or a paradox like the Growing Argument or the case of Achilles and the Tortoise. But if "becoming different from oneself" is as Simplicius describes it, then it is hard to see what is puzzling about it, and why it should be a philosophical *topos* worthy of a Sophist. There is a whiff of paradox in the phrase "becoming different from oneself,"[1] but to say that "the same Coriscus is sometimes in the market-place and sometimes in the Lyceum" seems entirely straightforward.

Second, the phrase ὃ ποτε ὄν, which I gloss as "with respect to its substrate" here, would seem to suggest that Coriscus, when in place *x*, is being thought of as the accidental unity Coriscus-in-place-*x*. This is because at *Parts of Animals* 2.3 649b20-7, which is the source of this standard gloss of the phrase ὃ ποτε ὄν, ὃ ποτε ὄν denotes the substrate of an accidental unity, viz. blood:

> These distinctions, then, being laid down, it is plain that blood is essentially hot in so far as that heat is connoted in its name; just as if boiling water were denoted by a single term, boiling would be connoted in that term. But the substratum of blood and that which it is while it is blood (τὸ δ' ὑποκείμενον καὶ ὅ ποτε ὂν αἷμά ἐστιν), is not hot. Blood then in a certain sense is essentially hot, and in another sense is not so. For heat is included in the definition of blood, just as whiteness is included in the definition of a white man, and so far therefore blood is essentially hot. But so far as blood becomes hot from some external influence, it is not hot essentially. (*PA* 2.3 649b20-7)

The claim, here, is that "blood" refers not to a distinct type of tissue, but merely to a hot phase of bodily fluid. "Blood" denotes an essence, but it is the essence of an accidental unity of the substrate bodily fluid and the property heat, just as "white man" denotes the accidental unity of the substrate man and the property whiteness. Bodily fluid is "that which blood is while it is blood," which means that bodily fluid may get hot and be called "blood," but it is still just bodily fluid.

There are two other places where Aristotle attributes a puzzle about Coriscus to the "sophists":

> The arguments of the sophists deal, we may say, above all with the accidental; e.g. the question whether "musical" and "lettered" are different or the same, and whether "musical Coriscus" and "Coriscus" are the same … *Metaph.* 6.2 1026b15-18

> Generally, if it is necessary to distinguish as the sophists do, [the good man] is related to himself as Coriscus to good Coriscus. For it is clear that some identical portion of them is good; for when they blame themselves, they kill themselves. *EE* 1240b24-27

Coriscus, like Socrates and Callias, is frequently used by Aristotle as a stock example of an individual human being, so we cannot put too much weight on the name alone. Still, I think that it is probably more than a coincidence that on each of these two occasions where a puzzle about Coriscus is attributed to the sophists, the puzzle seems to feature accidental unities like "musical Coriscus," and "good Coriscus." In the light of this, I think that the sophistical puzzle referred to at *Physics* 219b18-22 probably posits a difference between a plurality of accidental unities, viz. Coriscus-in-the-Lyceum and Coriscus-in-the-market-place, not of a single Coriscus *simpliciter* who is "sometimes in the market-place, sometimes in the Lyceum" as Simplicius suggests. At any rate, just after this passage, at *Physics* 220a2-4, Aristotle speaks of the "number of the changing thing," which would make no sense unless a changing thing like Coriscus were somehow a plurality that could be counted.

Accordingly, I think that the sophistical puzzle in question is probably raising a question similar to the one just quoted from *Metaphysics* book Ε, viz., whether Coriscus-in-the-market-place is indeed the same individual as Coriscus-in-the-Lyceum. And if the Sophists are suggesting that Coriscus-in-the-market-place becomes Coriscus-in-the-Lyceum, then he clearly does so by being replaced by Coriscus-in-the-Lyceum. And just as in the Growing Argument, a change of size results in a change of identity, so in the puzzle about Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum, a change of place results in the

emergence of a new individual. The simplest way to get this result is first to assume the indiscernibility of identicals, viz., if *x* is identical to *y*, then for every property *F*, object *x* has property *F* if and only if object *y* has property *F*. Then one supposes that Coriscus at time *t* has the property of being in the marketplace while Coriscus at time *t** does not, because Coriscus at time *t** is in the Lyceum. And then one concludes (by *modus tollens*) that Coriscus at time *t* cannot be identical to Coriscus at time *t**, because Coriscus at time *t* has a property that Coriscus at time *t** does not. Coriscus at time *t* and Coriscus at time *t** must be distinct individuals (viz., Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum respectively) that succeed each other in time.

2.1.2 Aristotle's solution

2.1.2.1 One in substrate, two in definition

The solution that Aristotle offers to this puzzle is that Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum are identical in one sense but not in another: "[The moving thing], with respect to its substrate, is the same, … but in definition it is different." The concept of being one in substrate but two in definition (or equivalently, two in form or in being) is one of Aristotle's favorite philosophical tools, and he deploys it to solve a range of problems that require an account of unity in diversity. It is applied to problems of change and persistence in *Generation and Corruption* 1.4-5 and *Physics* 1.7, as well as in the passage in *Physics* 4.11 that we have just seen. It is also used in *Physics* 3.3 and *De anima* 3.2 to claim that agency and patiency are unified in one sense but diverse in

another, and in *De anima* 2.12, 3.2, 3.7, and 3.9 to make a similar claim about the various faculties of the soul (e.g., the appetitive, the imaginative, etc.).

In *Physics* 3.3, Aristotle expands on this notion by giving an illustrative example. The road from Athens to Thebes is the same in substrate, i.e., numerically the same, as the road from Thebes to Athens, but it is not the same in being, form, or definition. Being, form, or definition, in this context at least, depends on the perspective from which these things are viewed, viz., the road from Athens to Thebes is the road from the point of view of one who is in Athens, and the road from Thebes to Athens is the road from the point of view of one who is in Thebes. The case is quite similar to Frege's example of the morning star and the evening star. The morning star is identical to the evening star, and the expressions "the morning star" and "the evening star" refer to the same object, but they do so from different points of view. "The morning star" means something like "the celestial object appearing in a certain region of the sky in the morning" and "the evening star" means something like "the celestial object appearing in a certain region of the sky in the evening." On this account, then, "Coriscus-in-the-Marketplace" and "Coriscus-in-the-Lyceum" are names for the same object but they pick out the object in different contexts or under different descriptions. Being-in-the-Marketplace and being-in-the-Lyceum are merely contexts or settings in which Coriscus appears. And this is not difficult to accept, since we would typically count place or location as part of the context in which a substance appears.

But Aristotle clearly wants intrinsic properties like color to be part of this context as well, and he establishes this point in *Physics* 1.7, in the course of solving another puzzle about change. The puzzle in *Physics* 1.7-8 is an Eleatic one, and it seeks to prove the impossibility of change based on the fact that change under certain descriptions seems to imply generation *ex nihilo*. It becomes clear, however, as Aristotle solves the Eleatic problem, that change under certain descriptions also seems to threaten the persistence of objects through changes. In Aristotle's treatment, the very same change is described in the following three sentences:

(i) The man becomes cultured.

(ii) The uncultured becomes cultured.

(iii) The uncultured man becomes the cultured man.

The key point, as regards the problem of persistence, is that sentences (ii) and (iii) seem to involve the replacement of the thing that changes by the thing it becomes. In sentence (ii), the uncultured disappears and is replaced by the cultured, and in sentence (iii), the uncultured man disappears and is replaced by the cultured man. (The uncultured man/cultured man pair is directly analogous to Coriscus-in-the-Marketplace/Coriscus-in-the-Lyceum, since they both represent accidental unities.) Sentence (i), however, does not give this impression, and it is therefore to be preferred. Sentence (i), in fact, allegedly reflects the underlying metaphysics of the change more accurately, since it shows that

> … there must always be an underlying something, namely that which becomes, and that this, though always one numerically, in form at least is not one. (By that I mean that it can be described in different ways.) For "to be man" is not the same as "to be uncultured." One part survives, the other

> does not: what is not an opposite survives (for "man" survives), but "not-cultured" or "uncultured" does not survive, nor does the compound of the two, namely "uncultured man. (*Phys*. 190a14-22)[2]

Hence, Aristotle's answer to the Coriscus-in-the-Marketplace/Coriscus-in-the-Lyceum puzzle may be generalized to claim that accidental properties (whether intrinsic or not) are to be taken as part of the context in which a substance appears. Properties like "cultured" and "uncultured," as well as accidental compounds that they form with substances (e.g., cultured man and uncultured man) come and go, but "man," or the entity that we properly regard as identical over time, does not. "Cultured Coriscus" and "uncultured Coriscus," as well as "Coriscus-in-the-Marketplace" and "Coriscus-in-the-Lyceum," just refer to Coriscus in different states of affairs or circumstances.

2.1.2.2 Further considerations

I imagine that a sophist who has received this reply to his puzzle of Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum might object that this is all well and good, but unless some further assumptions are spelled out, one cannot see how change can be possible. For even assuming that identity claims are only made about Coriscus *simpliciter*, one still needs to explain how it is possible for a single entity to feature in mutually exclusive states of affairs (like being in the marketplace and being in the Lyceum). In other words, what is it about the separation of these mutually exclusive states of affairs in time that allows them both to hold of the same individual without contradiction?

I can see one possibility if we assume that states of affairs are abstract entities or structured complexes consisting of one or more particular, one or more universal (i.e., a property, kind, or relation), and one or more exemplification relation between the particular(s) and the universal(s). If we make this assumption, we can claim that past and future states of affairs are uninstantiated. Past and future states of affairs do not conflict with present states of affairs, or with each other, because they do not exist.

2.1.2.2.1 Presentism?

What I am suggesting, of course, is presentism, which is the temporal analogue of possible worlds actualism. Just as the possible-worlds-actualist claims that only things that make up the actual world exist, the presentist claims that only things that exist presently exist. And just as the possible-world-actualist entertains possibilities about Coriscus by taking his existence in the actual world as given, and representing the same Coriscus counterfactually in non-actual possible states of affairs, the presentist represents Coriscus' past and future by taking his existence in the present as given, and thinking of the same Coriscus in states of affairs that no longer, or do not yet obtain.

Presentism, however, is not an attractive doctrine to ascribe to Aristotle. The most common objection to presentism, in the words of David Lewis, goes something like this: "No man, unless it be at the moment of his execution, believes that he has no future; still less does anyone believe that he has no past."[3] In other words, we all assume that the future and the past exist in *some* sense, and

this is reflected in the way we talk. So, it will not do simply to say that the past and future do not exist — full stop. Moreover, there are other reasons why Aristotle, in particular, would have a problem with presentism, which largely stem from his geometrical notion of the temporal continuum. Miller, for instance, points out that since time is an attribute of motion, and motion does not exist at an instant, time does not exist at an instant. Therefore, if Aristotle is to say that time is real, and he does, he must commit himself to the reality of either the past or the future.[4] Owen and Hussey make the slightly different point that the status of the now as a boundary of the past and future implies the reality of that which it bounds. Hussey thinks that this at least implies the reality of the past since, according to Aristotle, every moment of change is a moment of having changed.[5]

2.1.2.2.2 Serious tensing?

So presentism is an unattractive solution to our problem. But there are other approaches, and one of them is called "serious tensing" or "taking tense seriously."[6] The idea of serious tensing, basically, is that propositions are always to be evaluated from the perspective of some now. This means that any apparently tenseless proposition like the one expressed by "Coriscus at time *t** is in the Lyceum" must be translatable into a tensed proposition the tense of which is determined by the context of the (actual or imagined) assertion. If we choose to view the puzzle from the perspective of time *t,* for instance, then we must say that "Coriscus is now in the marketplace" and "Coriscus will be in the Lyceum." The

payoff, of course, is that these propositions may be asserted jointly without contradiction.

For my part, I find it implausible that a sentence like "Coriscus at time *t** is in the Lyceum" means anything like "Coriscus always was in the Lyceum at time *t*, Coriscus is in the Lyceum at time *t*, and Coriscus always will be in the Lyceum at time *t*." We typically say "Coriscus at time *t** is in the Lyceum" intending to make a tenseless claim, and I find it difficult to accept that this is in any way improper or incoherent. Perhaps a more promising strategy, which preserves the idea that propositions change their truth value but better accommodates sentences like "Coriscus at time *t** is in the Lyceum," is taking propositions to be true at times in the way that they are true at possible worlds. The idea is that at time *t,* the proposition expressed by the sentence "Coriscus is in the marketplace" will be true and the proposition expressed by the sentence "Coriscus is in the Lyceum" will be false, but these truth values will reverse at time *t**, so that it will not be the case that both propositions are true at any one time slice. So, to claim that Coriscus at time *t* is in the Marketplace and that Coriscus at time *t** is in the Lyceum is just to claim (tenselessly) that a certain proposition is true at time slice *t,* and another proposition is true at time slice *t**, but they do not conflict because they are not both true at any one time slice.

We find no such view elaborated in Aristotle, of course, but there is evidence that he thought that propositions changed their truth values over time. In *De interpretatione* 9, for instance, Aristotle claims that we can truly affirm the disjunction that either there will or will not be a sea battle tomorrow, but we

cannot truly affirm either of the disjuncts individually. Apparently, the proposition that there will be a sea battle tomorrow does not have a truth value until the sea battle takes place or fails to take place, at which time we may retrospectively attribute a truth value to this future-tensed proposition. I take it, then, that we may also talk tenselessly about a sea battle at time *t,* but if time *t* happens to be in the future, then the proposition "there is a sea battle at time *t*" will also lack a truth value, but will come to acquire one at time *t*.

Serious tensing and the view that propositions are true at times are approaches that focus primarily on propositions in attempting to solve the puzzle at issue. Clearly, however, some account needs to be given of the metaphysics of the truth makers of these propositions. Sally Haslanger has suggested that at the core of the serious tensing view is the doctrine that the having of properties is always tensed.[7] But this just tells us that to have a property is to have it in the past, present, or future. What does it mean, then, to have a property in the past, present, or future? If I am a presentist, to have a property in the present is to have it, full stop, while to have it in the past or future is not to have it at all. If I am an eternalist, and imagine the past and future to have a metaphysical standing that is comparable with the present, I have the property, but in a very odd sense. As Dean Zimmerman points out, I will still have yesterday's headache, but it will not be painful any more since it is a past headache.[8] But do we really want to admit ghostly entities like painless, non-present pains into our ontology?

2.1.2.2.3 Quasi-presentism: the future exists in potentiality

I think that neither of these options are either plausibly Aristotelian or very attractive philosophically. Fortunately, a middle way can be found, at least in the case of the reality of the future, that makes use of the analyses of Chapter 1. If the relation *x* is potentially$_{\beta}$ *y* orders the phases of Coriscus' movement from the marketplace to the Lyceum, then to have the property of being in the Lyceum in the future is the same as to be potentially$_{\beta}$ in the Lyceum. So, being prospectively in the Lyceum but currently in the marketplace is the same as being actually in the marketplace but potentially$_{\beta}$ in the Lyceum. Therefore, Coriscus' being in the Lyceum does not conflict with Coriscus' being in the marketplace because, at any time you care to choose, the way in which Coriscus has these properties differs. Now the possible worlds actualist would also say that Coriscus is possibly in the Lyceum when he is actually in the marketplace, but this is just a logical possibility. And in the presentist analogue of possible worlds actualism, there is no suggestion that futurity in any way reduces to possibility. Futurity and possibility are modalities the extensions of which happen to overlap in some instances (i.e., some possible states of affairs are also future states of affairs). But for Aristotle, as I have interpreted him, futurity is to be *identified* with potentiality in the sense of potentiality$_{\beta}$, and the future may be said to exist "in potentiality" in the same sense.

Now potentiality$_{\beta}$ is, I have argued, a special teleological potentiality, or a potentiality that has a characteristic "F-wardness" associated with it. The potentiality$_{\beta}$ of a stone to be at a lower altitude, for instance, is not the potentiality

for just one out of the many logically possible states of affairs the stone can enter into. It is a potentiality to enter into a state of affairs that is favored by the stone as its characteristic goal. And while its status as a goal might not fix it with certainty in the stone's future history, this possible state of affairs is nonetheless highly probable because the "innate impulse" of the stone is characteristically directed toward it. And if all of the world's natural substances have similar potentialities$_{\beta}$ and similar innate impulses to fulfill these potentialities, then this should confer on the future at least an etiolated form of existence. I suggest, then, that the future exists "in potentiality" in the potentiality$_{\beta}$ of natural things to realize their characteristic goals.

This interpretation of futurity also accords with our intuitions about what it means to "have a future." The man about to be executed has no future in the sense that he can no longer hope to realize any of his goals or wishes. The store of possibilities that stimulate and give content to his desires and aspirations have dwindled to nothing, and he has, literally, nothing to look forward to. To have a future, then, is just to have the potential to realize a goal or a wish.[9]

2.1.3 Criteria for identity

In summary, Aristotle's answer to the puzzle we have been considering is to claim that diachronic identity holds not between accidental unities like Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum but rather between instances of an "underlying something," viz., Coriscus, considered without his accidental properties. Accidental properties, in other words, are to be taken not as part of a

substance, but as part of the context in which a substance appears. Moreover, we saw that it is open to Aristotle to claim that in any case, the properties being-in-the-Marketplace and being-in-the-Lyceum do not conflict with each other because at any time when one of them is possessed actually, the other is possessed potentially. Yet there is still a difficulty that brings to mind Aristotle's assessment of his own initial response to one of Zeno's paradoxes, viz., "Although this solution is adequate as a reply to the questioner, … nevertheless as an account of the fact and explanation of its true nature it is inadequate" (*Phys*. 263a15-8). What Aristotle has done, essentially, is demonstrate that there is no contradiction in assuming that Coriscus *simpliciter* is identical over time. But he has given us no justification of this assumption. What is needed is a criterion for diachronic identity.

2.1.3.1 **x *is potentially*$_\beta$ y**

One possibility might be to think of the relation *x* is potentially$_\beta$ *y* as a criterion for diachronic identity. That is, if *x* and *y* appear in states of affairs that obtain at different times, and *x* is potentially$_\beta$ *y*, it follows that *x* is "identical in substratum" to *y*, since the relation connects a single entity in one state of affairs to *itself* in another state of affairs. For instance, the relation holds between Socrates-as-a-boy and Socrates-as-a-man and likewise between Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum. Accordingly, Socrates-as-a-boy and Socrates-as-a-man are identical in substratum, as are Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum. Since the "is" in the relation "*x* is potentially$_\beta$ *y*" is to be taken

tenselessly, this relation may be used to identify past, present, as well as future phases of a change.

At first sight, this approach has some intuitive appeal. Coriscus-as-a-boy has the potentiality$_{\beta}$ to become Coriscus-as-a-man, but not Socrates-as-a-man. Why? Because Coriscus' potentiality$_{\beta}$ is constrained somehow to prevent this. There are certain things Coriscus-as-a-boy has the potentiality$_{\beta}$ to become and Socrates-as-a-man is not one of them. Exactly why this is the case is not altogether clear.

2.1.3.2 Coriscus simpliciter*?*

But perhaps it will become clearer if we consider what it is that Coriscus *simpliciter* (i.e., the "underlying something" that remains identical from time to time) is supposed to be. Coriscus, like "the man" in the examples of *Physics* 1.7, is a substance. But in the *Metaphysics*, Aristotle tells us that there is a primary and a derivative sense of the word. In its primary sense, substance refers to "the form that is in the thing" while in a derivative sense, substance is "the whole combined from this and the matter" (*Metaph.* 1037a29-30). The question is, is it Coriscus' form that persists through changes, or is it the combination of Coriscus' form and matter?

Fortunately, there is a text that is quite specific that it is the form and not the form/matter composite that persists in living things, viz. *Generation and Corruption* 1.5. *Generation and Corruption* 1.5 claims that the form of a living organism persists through growth just as the shape of a tube persists when it is

inflated by water. And since *Generation and Corruption* 1.5 takes a form to be a shape or *morphê*, it is clear that it is a particular shape that is supposed to persist. *Physics* 1.7-9 might at first sight seem to clash with this conclusion by suggesting that matter also persists, but this just reflects the different aims of these passages. *Physics* 1.7 seeks to establish the general principle that an "underlying something" persists through every change, while *Generation and Corruption* 1.5 seeks to establish that this "underlying something" is the form in the case of biological growth. At the beginning of *Physics* 1.7, the focus is on alteration, and the "underlying something" is a "man," Aristotle's paradigm example of a substance. But then at 190a31 ff., Aristotle broadens his point to include substantial change, so that the "underlying something" can be either matter or a substance, or both, depending on the context. For instance, at 190b13-17, he refers indifferently to substances and to matter as "that which underlies": "By that which is opposed, I mean the ignorant of music, by that which underlies, the man; and shapelessness, formlessness, disarray are opposed, the bronze, the stone, the gold underlie." And similarly at 190b23-27: "The underlying thing, however, though one in number, is two in form. On the other hand there is the man, the gold, and in general the measurable matter; this is more of a this thing here, and it is not by virtue of concurrence that the thing which comes to be comes to be from this." Thus, a consistent reading of *Physics* 1.7 and *Generation and Corruption* 1.5 would be that the substance understood as the form persists through growth and alteration, while the matter persists through substantial change.

There are also strong philosophical reasons why it should be the form and not the form/matter composite that persists, and that the form should be an individual form. For if the form/matter composite is supposed to have the same form and the same matter from time to time, it will run up against the common (and I think correct) objection that the matter of a form/matter composite is always changing. This is the assumption behind the Growing Argument, which dates to Epicharmus in the 5th century B.C.E., and which would have posed a challenge to anyone who wanted to make material composition part of their account of persistence. But if it is the form of Coriscus that remains the same through a change, the form cannot be a universal. To see why this is so, imagine that the form *were* a universal, so that the form of Coriscus and the form of Socrates are not individuated in any way, and so that it is the matter that individuates the form/matter composite at any given point in time. At time *t,* for instance, it would be the matter of each that makes Coriscus and Socrates the individuals they are, while their form makes them the *type* of thing they are. But if the matter of Coriscus and Socrates changes between time *t* and time *t*,* then they cannot be re-identified as the *same* individuals they were at time *t,* since their identity as individuals was determined by their material composition at time *t.* At best, they can be reidentified as the same *type* of individual because they have the same universal form. Hence, in order to underwrite a doctrine of persisting individual substances, Aristotle would at least need to envisage individual instances of type-identical species forms.

2.1.3.3 Spatio-temporal histories

Since it appears to be the *individual* form of Coriscus that persists through a change, our criterion of diachronic identity will clearly need to be some feature of this form. But Michael Frede points out that if individual forms are type-identical, and the criterion is imagined to serve as a criterion for both synchronic and diachronic identity (because, as I have argued above, Coriscus and Socrates are to be diachronically reidentified as the same *individuals*, not as just the same type of individual), then it cannot be any feature of the species essence that is our criterion, since these features are shared by other members of the species. Frede suggests that the "feature" of individual forms we are looking for is their spatio-temporal histories.[10] In particular, he claims that it is the *continuity* of a form's spatio-temporal history that secures its diachronic identity. What I take this to mean is that an individual form is identical over time if there are no gaps in its existence, and if the form's instantiations at different times are "connected" by a single continuous world line. Aristotle would, no doubt, accept this sort of spatio-temporal continuity as at least a necessary condition for diachronic identity. If a form is thought of as a particular organization, disposition, or structure, then it bears a similarity to a *hexis*, like health. And at *Physics* 5.2, 228a6-12, Aristotle seems to rule out the possibility of *hexeis* like health returning to existence after a period of non-existence (i.e., sickness).[11]

But this seems rather arbitrary. Surely there is nothing logically incoherent about supposing that the same individual form could be instantiated at two times without a continuous world line connecting them. In fact, Derek Parfit

has concocted a science fiction scenario that envisages just this.[12] He imagines that at some future time, a teletransportation device is developed that will record the "exact states of all [one's] cells" and transmit the information to another device on Mars. The latter device will then create a brain and body on Mars with exactly the same psychological and physical states, and the former device will destroy the brain and body on earth. Parfit's claim is that the individual who walks into the machine on Earth is the same individual who steps out of the machine on Mars. Now if the individual on Earth and the individual on Mars have exactly the same psychological and physical states, then I would say that they probably have the same Aristotelian form, and since, according to Parfit, it takes radio waves about three minutes to travel from Earth to Mars, there is a discontinuity of both place and time in the existence of the form.[13]

2.1.3.3.1 Time-indexed properties

Now clearly, Aristotle would not have thought such a scenario to be within the realm of physical possibility, and he would not have taken the care to accommodate it. But I think that a more robust criterion of identity is at hand that would accommodate Parfit's scenario, and indeed, one that is no less plausible than the spatio-temporal continuity of a world line, viz., the spatio-temporal history itself. Let us imagine, for instance, that Coriscus has the property of being in the marketplace at time *t*, and Coriscus has the property of being in the Lyceum at time *t**. This is just part of Coriscus' spatio-temporal history. But as I have described it, it is not yet a unique history, and, therefore, could not be used as a

way to pick out a unique individual at different times. Coriscus might have been taking a walk with Socrates, which entails that Socrates too was in the marketplace at time *t* and in the Lyceum at time *t**. So let's be more specific. Let's say that Coriscus gets a drink at the Enneacrounos at time *t* (a well at the southeast corner of the Agora) and that he is the only one that is doing this at time *t*. I claim that Coriscus at time *t* is identical to Coriscus at time *t** if and only if Coriscus at time *t** has the property of having-been-at-the-Enneacrounos-at-time-*t*.

My idea is simple. To have a spatio-temporal history is just to have a collection of time-indexed properties. To have a unique spatio-temporal history is just to have at least one property contained in that history that is not shared with any other individual. What is attractive about this proposal is that at one stroke it provides a criterion for both individuation and identity over time. If Socrates and Coriscus have type-identical species forms that are spatially distinct at any point in their history, then Socrates and Coriscus are distinct not only at that point in time, but as long as they exist, since their histories will be distinct as long as they exist. And for this reason, they will be able to be reidentified as the individuals with these distinct histories. This makes spatio-temporal histories essential properties, not in the usual Aristotelian sense of the word, which has to do with what it is to be something, but in the sense of simply a property that a thing has in every circumstance in which it exists.

Time-indexed properties are close analogues to world-indexed properties, which Plantinga uses to secure the identity of individuals across possible worlds.

For instance, if person *x* has the property *F-in-*W_α of standing at the corner of Hollywood and Vine in possible world W_α, or the property *F-at-t* of standing at the same place at time *t*, he or she has the property *F-in-*W_α in all possible worlds or the property *F-at-t* for all time. Moreover, if person *x* is the *only* person standing at the corner of Hollywood and Vine in possible world W_α or at time *t*, then he or she will be the only person in any possible world or at any time that has this property.[14]

Admitting time-indexed properties like this does not, of course, commit us to the view that *all* properties are time indexed. The claim is only that properties that are part of one's spatio-temporal history are time indexed, and that one can "just have" the property of having-been-at-the-Enneacrounos-at-time-*t* just as one might "just have" the property of being pale. Similarly, one can acquire the property of having-been-at-the-Enneacrounos-at-time-*t* just as one might acquire the property of being pale. The purpose of time-indexing properties here is not to allow them to be timelessly (and tenselessly) predicated of a persisting subject, but to reflect the fact that they are part of a spatio-temporal history.

2.1.3.3.2 An objection

On the view that I have been advocating, one might think that a problem arises of the following sort: Remember that the puzzle of Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum was solved by distinguishing between Coriscus *simpliciter*, who is diachronically identical, and certain accidental unities like Coriscus-in-the-Marketplace and Coriscus-in-the-Lyceum, which are not. In

effect, Aristotle stipulated that accidental properties are to be taken as part of the context in which a substance appears. We also saw that Coriscus *simpliciter*, if we take him to be analogous to "the man" in the analysis of *Physics* 1.7, could be either the form of Coriscus or the form of Coriscus combined with his matter. But we found strong philosophical reasons, as well as textual support in *Generation and Corruption* 1.5, to prefer Coriscus as a form, and not as a form/matter compound, because the matter in a form/matter compound is always changing. So, Coriscus *simpliciter* is the form of Coriscus, and it is *this* that Aristotle exempts from accidental change, in order to claim that it is identical over time.

But now we want to claim that a property like having-been-at-the-Enneacrounos-at-time-*t* is a property of Coriscus' form, and, indeed, a property that serves as the basis for individuating it as well as reidentifying it at different times. And if Coriscus' form can acquire such a property (presumably, it can never lose it), then Coriscus *simpliciter* will fail to be indiscernible, and therefore identical, before and after the property is acquired.

One strategy that comes to mind is for Aristotle to claim that the time-indexed properties in a spatio-temporal history are held tenselessly, so that they are never gained or lost. Yet, as I pointed out in my discussion of serious tensing above, Aristotle *does* believe that propositions change their truth values as time passes. Thus, if time *t* is in the future, "there is a sea battle at time *t*" will have no truth value, but will come to acquire one as time *t* passes. If this is because there is no fact of the matter regarding the sea battle until it happens, then the property

being-in-a-sea-fight-at-time-t cannot be possessed before time *t* and must, therefore, be acquired at time *t*.

But even if we must accept the fact that time-indexed properties can be acquired, we would do well to consider the nature of properties like having-been-at-the-marketplace-at-time-*t*. Previously, we thought it reasonable that Coriscus *simpliciter* remains the same throughout his perambulations because "Coriscus-in-the-Marketplace" and "Coriscus-in-the-Lyceum" just refer to him in two different states of affairs or circumstances. Just as "Hesperus" and "Phosphorus" refer to the same object, but with a different Fregean sense, so too do "Coriscus-in-the-Marketplace" and "Coriscus-in-the-Lyceum." But would it not at least be curious if the property of being-in-the-marketplace, which when possessed in the present tense is taken merely to be part of the context in which Coriscus appears, when possessed in the past tense becomes part of Coriscus himself? In other words, why should a history of events of a certain sort be thought to alter an object when the events themselves do not?

Or consider the implication of generalizing Aristotle's solution to the Coriscus-in-the-Marketplace/Coriscus-in-the-Lyceum puzzle to claim that *all* accidental properties (whether intrinsic or not) are to be treated like being-in-the-marketplace and being-in-the-Lyceum. I mean, if intrinsic as well as relational properties are to be treated as part of the context in which Coriscus *simpliciter* appears, then intrinsic properties are being demoted to the level of relational properties for the purposes of the analysis. This, I think, is the rationale for thinking that the acquisition or loss of an intrinsic property should not impact the

identity of Coriscus *simpliciter*. But if regarding properties like these as on par with relational properties makes them etiolated enough to regard them as part of the context in which a substance appears, then their status as *past* relational properties should make them even more etiolated.

2.1.3.3.3 Objection answered/The reality of the past

The most reasonable approach, I think, is to suppose that properties like having-been-in-the-marketplace-at-time-*t* are etiolated enough to regard them as part of the context in which a substance appears, but not so etiolated as to be incapable of serving as criteria of synchronic and diachronic identity for the forms in question. This accords, I think, with our intuition that the past should be real, but not quite so real as the present. If having-been-in-the-marketplace-at-time-*t* is a real property of Coriscus that serves to individuate and reidentify him at every moment thereafter, then time *t,* in a sense, lives on in the person of Coriscus as well as in every extant entity that existed at that time. But, as is the case with properties like being-prospectively-in-the-marketplace, it does not exist on a comparable metaphysical footing with properties like being-(presently)-in-the-marketplace. Properties like being-prospectively-in-the-marketplace are etiolated because they are potential properties. Properties like having-been-in-the-marketplace are etiolated because they are historical properties. And it is this etiolation that allows mutually exclusive past and future states of affairs to hold of the same individual without contradiction.

2.2 Growth: The Growing Argument

2.2.1 The puzzle

Another, and much more famous ancient puzzle about persistence is the Growing Argument. Although the name "Growing Argument" (αὐξανόμενος λόγος) was coined by the Academics in the 3rd-century BCE, the argument itself originated with the 5th-century BCE comic playwright Epicharmus, and it survives in the following fragment:

> DEBTOR. If you like to add a pebble to an odd number—or to an even one if you like—or if you take one away that is there, do you think it is still the same number? CREDITOR. Of course not. D. And if you like to add some further length to a yard-measure, or to cut something off from what's already there, will that measure still remain? C. No. D. Well, consider men in this way too—for one is growing, one declining, and all are changing all the time. And what changes by nature, and never remains in the same state, will be something different from what changed; and by the same argument you and I are different yesterday, and different now, and will be different again—and we are never the same. (23 B 2 Diels-Kranz)[15]

The argument turns on the assumption that the personal identity of an individual is a strict function of its material composition. Since the material composition of our bodies, so the argument goes, is in a state of constant flux, and since our identities are a strict function of this material composition, our personal identities are also in constant flux.

2.2.2 Aristotle's solution

While Plato mentions Epicharmus as a flux theorist at *Theaetetus* 152e, Aristotle makes no explicit mention of either Epicharmus or his puzzle. Still, Aristotle's

account of biological growth in *Generation and Corruption* 1.5 shows a sensitivity to the issues raised in the puzzle and develops a way to deal with them. As in the Growing Argument,[16] Aristotle takes biological growth and diminution to take place through the accrual and loss of matter to and from the growing substance, and he devises a way for individuals to persist in the face of this fact. But unlike the Growing Argument, Aristotle also takes it as a datum that biological growth means growth in every part of the organism at once. This, however, is thought to pose an additional problem, since if growth were to occur by accrual to *every* part of the organism at once, then the accruing matter would need to pass through other matter as it disperses uniformly throughout the body, and this is thought to be impossible. Aristotle's solution is a typical Aristotelian compromise, one that meets all of his desiderata by distinguishing a way in which growth takes place, viz., growth in the form, and a way in which it does not, viz., growth in the matter. Matter increases in bulk, but it does not grow in the relevant sense because it does not grow in every part. Growth in the form, however, *does* occur in every part, but not by the accrual of matter to every part. Aristotle did not have rubber balloons at hand, but he seems to imagine that biological organisms grow as balloons do when they are filled with water, where the balloon represents the form and the water represents the matter. The volume of water can increase by the in-flow of new water, but the new water need not disperse uniformly throughout the balloon to effect an expansion of the balloon. Since the balloon gets larger while retaining the same shape, the form, as the shape of the balloon, persists. Aristotle talked of a tube (*aulos*) instead of a balloon.

Alexander and Philoponus suggest that the tube is to be expanded by water running through it, in much the same way as I have suggested with the balloon.[17] Here is the passage from Aristotle:

> This form, like a tube (*aulos*), is a force (*dunamis*) in matter. If some matter comes in which is potentially a tube, with its quantity potential too, these [combined] tubes will be larger. But if [the matter] can no longer act, but is like water mixed with wine in ever greater amount which eventually makes the wine watery or water, then it will produce a wasting away. The form, however, persists. (*GC* 322a28-33)

Sorabji points out that the "form" likened to a tube, here, is quite unlike the form equated with the soul in *De anima* 2.1.[18] In *De anima* 2.1, the soul as the form of the body is defined as "the first actuality of a natural body which has life potentially" (*DA* 412a27-8). A first actuality is a *hexis*, or a state or disposition for certain characteristic animate activities. The form likened to a tube, however, is a *morphê*, or a shape. In *Generation and Corruption* 1.5, it is the form as a *morphê* or a shape that persists through the change, viz., it is the same shape, just bigger.

Now it may appear that Aristotle has made a mistake, here. It is the mistake that we had thought that we had spotted with respect to time-indexed properties. In that case, we were able to acquit Aristotle, by claiming that the properties featured in the history of a form are etiolated enough to regard them as part of the context in which a substance appears, but not so etiolated as to be incapable of serving as criteria of synchronic and diachronic identity. The mistake would seem to be that in *Generation and Corruption* 1.5, Aristotle also has the form undergoing a change, but it is a change that is difficult to see as being in any way etiolated. That is, Aristotle makes the form itself grow, and if

this is permitted, it is difficult to see how the form will not be discernibly different before and after the change. And if the form of Coriscus-as-a-boy actually gets bigger when he becomes Coriscus-as-a-man, and Coriscus *simpliciter* is Coriscus' form, then Coriscus *simpliciter* will not remain diachronically identical through the change. In the case of inflating a balloon, we say that a spherical balloon retains the same shape as we blow it up. But surely, we mean the same *kind* of shape, because token shapes have sizes, and clearly, the size of the balloon is larger after it is inflated. Similarly, if the form of Coriscus-as-a-man is larger than the form of Coriscus-as-a-boy, then the forms are only the same in kind, not in number.

A possible way out of this is for Aristotle to use the relation *x* is potentially$_{\beta}$ *y* as a criterion for diachronic identity, as I suggested in Section 2.1.3.1 above. In this case, the form of Coriscus-as-a-man is identical with the form of Coriscus-as-a-boy, even though it is bigger, because the form of Coriscus-as-a-boy is potentially$_{\beta}$ the form of Coriscus-as-a-man. And as I suggested above, this identity could, in turn, be justified by an appeal to time-indexed properties possessed by the growing form. The form of Coriscus-as-a-man is identical with the form of Coriscus-as-a-boy, even though it is bigger, because they share a unique time-indexed property in their spatio-temporal histories.

2.2.3 Chrysippus' solution

Another ancient response to the problems raised by the Growing Argument is the one attributed to the Stoic philosopher Chrysippus by Philo of Alexandria, in his

De aeternitate mundi 48 (*SVF* II 397). Chrysippus, unlike Aristotle, responds directly to the Growing Argument, and his response is interestingly different. For he not only provides a solution to the problem of persistence-in-flux, he also mounts an attack on the legitimacy of the problem itself. The passage from Philo reads as follows:

> (1) Chrysippus, the most distinguished member of their school, in his work *On the Growing [Argument]*, creates a freak of the following kind. (2) Having first established that it is impossible for two peculiarly qualified individuals (δύο ἰδίως ποιούς) to occupy the same substance jointly, (3) he says: "For the sake of argument, let one individual (τὸν μέν) be thought of as whole-limbed, the other (τὸν δέ) as minus one foot. Let the whole-limbed one be called Dion, the defective one Theon. Then let one of Dion's feet be amputated." (4) The question arises which one of them has perished, and his claim is that Theon is the stronger candidate. (5) These are the words of a paradox-monger rather than of a speaker of truth. For how can it be that Theon, who has had no part chopped off, has been snatched away, while Dion, whose foot has been amputated, has not perished? (6) "Necessarily," says Chrysippus. "For Dion, the one whose foot has been cut off, has collapsed into the defective substance of Theon. And two peculiarly qualified individuals cannot occupy the same substrate. Therefore it is necessary that Dion remains while Theon has perished."[19]

Perhaps the most widely accepted interpretation of this passage is that of David Sedley in his 1982 article "The Stoic Criterion of Identity."[20] In this chapter, I will offer an interpretation that leaves intact the most important features of Sedley's account, chief among which is his view on the basic purpose of the puzzle. Like Sedley, I take the fact that the puzzle appears in a work called *On the Growing Argument* to indicate that it is a rejoinder to, and indeed, a *reductio ad absurdum* of the Growing Argument. Where I diverge from Sedley's approach, I do so to shore it up against certain objections to which I think it is vulnerable. My

chief concerns are to achieve a better fit with the text and to ensure that, since we view the puzzle as a *reductio ad absurdum*, we do not take Chrysippus to be deducing a contradiction by means of premises extrinsic to the Growing Argument. Otherwise, Chrysippus' *reductio ad absurdum* would fail in its purpose to show that the Growing Argument is *internally* inconsistent.[21]

I also follow Sedley on two other significant interpretive points. First, I agree that since, from at least Chrysippus' point of view, the puzzle runs up against the principle that "two peculiarly qualified individuals cannot occupy the same substrate," we *must* suppose that we are dealing with one body at the outset and that Theon is a part of Dion. Otherwise, when the foot is chopped off, the resulting state of affairs would not run up against this principle, and it is apparent from the text that it must. Besides, as Sedley also points out, Philo essentially tells us that Theon is a part of Dion several pages later in the same text (Philo, *Aet*. 49-51). Prior to Sedley's 1982 article, the consensus was "that [Dion and Theon] are supposed to be two numerically distinct individuals who are qualitatively identical except for the fact that Theon has a foot missing: hence when Dion's foot is amputated the two are made completely indistinguishable…"[22] Second, the justification for Dion's survival Sedley supplies on behalf of Chrysippus seems right. The amputee who is grieving over his severed foot must be Dion since "Theon cannot have lost a foot that was never part of him in the first place."[23]

Here is a very preliminary paraphrase of how Chrysippus' argument appears to run that incorporates these points. At the outset we have one living, anatomically complete human being named Dion, a region of whose body has

been named Theon – the whole body except one of its feet. The foot just mentioned is then amputated, with the result that either Dion or Theon must perish because, as Chrysippus tells us (and as Philo apparently agrees), "two peculiarly qualified individuals cannot occupy the same substrate."[24] A dispute arises about who should perish. Chrysippus claims that Dion should survive and Theon should perish, since it cannot be Theon who is grieving over his severed foot. But Philo claims, on behalf of the Academics, that Theon must survive and Dion perish, "for how can it be that Theon, who has had no part chopped off, has been snatched away, while Dion, whose foot has been amputated, has not perished?" I will argue in the sequel that the result favored by Philo is congenial to what the Growing Argument would predict – that Theon should survive and Dion should perish – while the result favored by Chrysippus is not. This, I believe, supports Sedley's claim that Chrysippus' puzzle is a *reductio ad absurdum* of the Growing Argument.

2.2.3.1 An interpretation of Chrysippus' puzzle

If, as seems probable, it was the Academy of Arcesilaus that revived Epicharmus' puzzle in the third century BCE, then it seems likely that the Academics meant it to be a *reductio ad absurdum* of the very notion of personal identity.[25] The conclusion that our identities are in constant flux obviously conflicts with the common sense view that personal identity is continuous over time. And given Plutarch's testimony that the Academy "suspended judgment about everything" (Plutarch, *Against Colotes* 1120C, 1121E-1122A), we should probably assume

that the Academic version of the Growing Argument is meant to be *aporetic*; that is, instead of taking the Academics to be committed to one or the other of the conflicting views – either that matter is the sole principle of identity, or that identity is continuous over time – we should take them to be exposing a conflict between these views and then suspending judgment about its resolution. In light of this, then, if Chrysippus' puzzle is itself a *reductio ad absurdum*, it is a *reductio ad absurdum* of a *reductio ad absurdum*, where Chrysippus exposes *unintended* absurdities in the Academics' Growing Argument.

Sedley says that the target of Chrysippus' *reductio ad absurdum* is the Growing Argument's assumption that matter is the sole principle of identity[26] – that the personal identity of an individual is a strict function of its material composition. Even though no such principle is expressed in the puzzle, this view makes good sense of a premise that would otherwise be quite baffling – the fact that Theon and Dion are apparently related to each other as part to whole.[27] Chrysippus' *reductio ad absurdum* comprises, basically, a main argument with a corollary. By reducing to absurdity the premise that Theon and Dion are related as part to whole, it also reduces to absurdity the assumption that matter is the sole principle of identity, because the former is validly deduced from the latter. Thus, Sedley says that Chrysippus "borrows from the Growing Argument's own presuppositions" to "concoct" a premise in which Theon and Dion are related to each other as part to whole. "According to the Growing Argument," he says, "every material addition to or subtraction from an individual results in his replacement by a new individual; and since in such cases the old and the new

individual are related as part to whole or whole to part, the Academic argument does indeed imply that whole and part constitute distinct individuals – the very premise that Chrysippus' own paradox presupposes."[28] The material additions and subtractions Sedley has in mind are, no doubt, the changes in bodily bulk caused by the ingestion of food and excretion. I think, however, that Chrysippus has something a bit more bizarre in mind.[29] We are meant, first, to imagine an individual named Theon, who happens to lack a foot. Then we suppose that for a certain restricted period, Theon's body only experiences one material (and quite miraculous) fluctuation – he grows a new foot. According to the Growing Argument, since the material composition of Theon has changed, we now have a new individual. Let us call him Dion. But since the personal identity of each individual is a strict function of its material composition, and since all of the flesh that constituted Theon is still present in a particular region of the individual that we now call Dion, we must still view this region of Dion as a numerically distinct individual that is related to Dion as part to whole. Therefore, Theon is related to Dion as part to whole.

But at first sight, there appears to be a problem. Although this seems to be a valid deduction from the principle that personal identity is a strict function of material composition, *prima facie* it is in direct conflict with the conclusion of the Growing Argument that growth is actually "generation" and "destruction." According to Plutarch, the Growing Argument concludes that "the prevailing convention is wrong to call these [material fluctuations] processes of growth and decay: rather they should be called generation and destruction, since they

transform the thing from what it is into something else" (*Comm. not.* 1083A-1084A). Likewise, the Epicharmus fragment concludes that as a man grows, his former self "withers" (23 B 2 Diels-Kranz). Therefore, since old individuals allegedly "wither" when new individuals come into being as a result of growth, Theon should have perished when he grew the foot, rather than becoming part of Dion. Moreover, Plutarch, who is a spokesman for the Academics, seems to think that the notion of two people being in one body is ridiculous. Plutarch, in fact, criticizes the Stoic notion of the peculiarly qualified individual precisely because he says it implies the view that each of us is composed of a multiplicity of entities — a parcel of matter, and a peculiarly qualified individual. For comic effect, Plutarch even likens the Stoics to Pentheus, the deranged king of Thebes, who in seeing double was "going crazy in his arithmetic" (*Comm. not.* 1083a-1084a).

These considerations raise two questions. First, are there resources within the Growing Argument, as it is transmitted to us, to resist the conclusion that Theon is a living part of Dion? If there are, then we will clearly need to rethink our position that Theon is a living part of Dion and perhaps even our claim that Chrysippus' puzzle is a *reductio ad absurdum*. The reason, of course, is that we cannot imagine a *reductio ad absurdum* of the Growing Argument to include premises that no proponent of that argument would accept. Second, if there are no such resources and the contradiction we have just discussed is unavoidable, can one of the conflicting claims be rejected, and if so, which one? Can one reject the contention that growth is actually "generation" and "destruction" or must one reject the conclusion that Theon is a living part of Dion?

These questions can only be answered by taking a closer look at the texts. The Epicharmus fragment frames the argument as follows:

> DEBTOR. If you like to add a pebble to an odd number—or to an even one if you like—or if you take one away that is there, do you think it is still the same number? CREDITOR. Of course not. D. And if you like to add some further length to a yard-measure, or to cut something off from what's already there, will that measure still remain? C. No. D. Well, consider men in this way too—for one is growing, one declining, and all are changing all the time" (23 B 2 Diels-Kranz).[30]

It seems fairly clear that this version of the Growing Argument permits the same inference that allows Sedley to conclude that Theon is a part of Dion. If I add one pebble to a set of, say, eight pebbles, the number of pebbles would now be nine, but the original eight pebbles would still be present as a subset of the new total. Plutarch's most extended description of the Growing Argument seems to allow precisely the same inference. Plutarch lists the premises of the argument as follows: "all particular substances are in flux and motion, releasing some things from themselves and receiving others which reach them from elsewhere; the numbers or quantities which these are added to or subtracted from do not remain the same but become different as the aforementioned arrivals and departures cause the substance to be transformed." And from these premises Plutarch concludes that "the prevailing convention is wrong to call these [material fluctuations] processes of growth and decay: rather they should be called generation and destruction, since they transform the thing from what it is into something else" (*Comm. not.* 1083a-1084a).[31] It seems clear that if we imagine a case in which something "receives some things from elsewhere" while not at the same time "releasing some things from itself," nothing in Plutarch's account

would block the inference that "the old and the new individual are related as part to whole or whole to part." Just as in the Epicharmus fragment, every change in the "number or quantity" of material parts in an individual results in a change in its identity.[32] And if growth is simply the augmentation of an existing set of material parts, then clearly the unaugmented set will persist as a subset of the augmented set. Granted, an Academic might insist that an individual must be a discrete body, which would defeat the line of argument I am attributing to Chrysippus. But this would amount to introducing a new premise that appears nowhere in our sources and does not strictly follow from the view that matter is the sole principle of identity. Indeed, one could imagine Chrysippus' answer to the suggestion that individuals must be discrete: "Oh, so material composition is not the sole principle of identity after all. Now it seems to be material composition *and* discreteness." If the Growing Argument did not contain the requirement of discreteness, then there would have been no reason for Chrysippus to recognize it in his *On the Growing Argument*. The Academics may well claim, in a rejoinder to Chrysippus, that an individual must be a discrete body, but this should have no effect on how we interpret the text at hand.[33] One might also object that since the Growing Argument envisages diminution, the set/subset relationship we have been considering would be disrupted when diminution occurs at the same time as growth. But the Growing Argument does not say that growth and diminution acting in concert constitute generation and destruction. Rather, the claim is that growth and diminution each constitute *both* generation and destruction, and for this reason it is perfectly legitimate to consider the case

of growth in isolation. This feature of the Growing argument is quite clear in the Epicharmus fragment. Whether Epicharmus is describing addition or subtraction, the alleged outcome is the same: the old number perishes when the new one comes into being. Plutarch's language is more ambiguous, but since he ascribes his argument to Epicharmus without signaling any disagreement, I see no reason to interpret Plutarch's account of the Growing Argument differently. When we consider the case of growth in isolation, there is nothing in the Growing Argument to block the inference from the assumption that matter is the sole principle of identity to the possibility that Theon could be a part of Dion.

Since this is the case, and a contradiction is unavoidable between this result and the view that growth is actually "generation" and "destruction," we can now turn to the question of which of the two, if either, can be rejected. The fact that the claim that growth is actually "generation" and "destruction" appears as a *conclusion* of the Growing Argument helps us here. That is, the foregoing analysis seems to show us that Plutarch and Epicharmus are wrong to claim that the premises of their argument establish that growth is actually "generation" and "destruction." The supersession of successive individuals undergoing growth results in the *incorporation* of the superseded individuals instead of their destruction. Thus, it appears that one must reject the claim that growth is actually "generation" and "destruction" because it has not been validly inferred from the premises of the Growing Argument. This result, I think, implies that Sedley's view that Theon is a living part of Dion need only be modified to recognize that Chrysippus must have undertaken a certain sort of argument in *On the Growing*

Argument prior to the passage Philo summarizes – one that that convicts the Growing Argument of the logical error I have just described, and *forces* this premise on the Academics against their will.

2.2.3.2 Interpretative problems

There also appears to be a problem with including, in a *reductio ad absurdum* of the Growing Argument, the premise that "two peculiarly qualified individuals cannot occupy the same substrate." I doubt Sedley's claim, that a proponent of the Growing Argument would accept this as a "common sense principle," chiefly because I find it incredible that the Academics would even acknowledge, much less think it common sense, that there is such a thing as a peculiarly qualified individual. This is because the "peculiarly qualified individual" was a Stoic invention intended to neutralize their own (the Academics') Growing Argument. Sedley says that Plutarch, arguing on behalf of the Academics, implicitly accepts the existence of peculiarly qualified individuals in his treatise "On Common Conceptions." But if we consider the nature of the cited passage, it seems that this cannot be true. The passage Sedley refers to (*Comm. not*. 1077C-E) is itself a *reductio ad absurdum* of another Stoic doctrine – that Zeus and Providence come to occupy the same aether during the Conflagration. Arguing on behalf of the Academics, Plutarch supposes that Zeus and Providence are peculiarly qualified individuals, so that he can draw the unwelcome conclusion for the Stoics that their story about Zeus and Providence requires two peculiarly qualified individuals to occupy the same substrate. As in any *reductio*, Plutarch entertains premises he

need not accept – that peculiarly qualified individuals exist – in order to bring out inconsistencies in a contested Stoic theory. Plutarch, in fact, seems to think that the idea of a peculiarly qualified individual is manifestly absurd. For as I pointed out above, it implies a claim that is non-evident, if not obviously false, viz., that each of us is composed of a multiplicity of entities. It is clear from this that Plutarch does not countenance the existence of peculiarly qualified individuals, and it is even clearer that he would not think that any proposition about them could qualify as common sense.

So, how should we view the premise that "two peculiarly qualified individuals cannot occupy the same substrate" in light of this difficulty? I think this obstacle can be overcome as long as we consider that the very definition of personal identity is in dispute. When a Stoic or a proponent of the Growing Argument confronts a puzzle like this, each will construe the term "individual" according to his own definition (granted, of course, that an Academic would take such a definition dialectically). Consequently, we must keep in mind two points of view as we run through the argument: The Growing Argument defines the individual as a particular collection of material parts. Thus, when a proponent of the Growing Argument is told that Dion and Theon are individuals he will argue that they are collections of material parts. The Stoics, on the other hand, hold that if Dion and Theon are individuals, they must be peculiarly qualified individuals. So they, of course, will think of Dion and Theon as such when they consider the puzzle. It is important to note that Philo does not explicitly state that Dion and Theon are peculiarly qualified individuals. And it is also telling that he reports in

indirect discourse that "it is impossible for two peculiarly qualified individuals (δύο ἰδίως ποιούς) to occupy the same substance jointly" but then switches to a direct quotation as follows: "[Chrysippus] says, 'For the sake of argument, let one individual (τὸν μέν) be thought of as whole-limbed, the other (τὸν δέ) as minus one foot.'" This leads me to suspect that the two premises just stated are not part of a continuous quotation, and that the τὸν μέν and τὸν δέ in line 3 need not refer back to the δύο ἰδίως ποιούς in line 2. Certainly, at the end of the passage Chrysippus does say that one of the two must perish *because* two peculiarly qualified individuals cannot occupy the same substrate. But this just reflects the Stoic diagnosis of the problem, and there is nothing to prevent a proponent of the Growing Argument from interpreting this stipulation in an entirely different way – that two (not necessarily peculiarly qualified) individuals cannot occupy the same substrate.

We can also take comfort in the fact that Chrysippus' *reductio ad absurdum* still works, even if we assume that Dion and Theon are not peculiarly qualified individuals: When Dion's foot is amputated, the Growing Argument requires that we call the amputee Theon, because we again have the same collection of flesh that we initially attached this name to. But, as Sedley suggests, there is a good *prima facie* reason to call the amputee Dion. For why would Theon be grieving over a foot he never had? Thus, the Growing Argument says that the amputee is Theon, but common sense says that it is Dion. The amputee cannot be both Dion and Theon because of the principle that two individuals cannot share all of their material parts. So since Dion is alive, then Theon must be

dead just as Chrysippus claims, and the Growing Argument is contradicted without making use of any propositions about peculiarly qualified individuals.

At first sight, it seems somewhat puzzling that the Academics would accept the stipulation that "two (not necessarily peculiarly qualified) individuals cannot occupy the same substrate," since they might still have escaped the conclusion that Theon is dead by saying that the amputee is *both* Dion and Theon – that Dion and Theon are still numerically distinct individuals, but their spatio-temporal histories have converged. The stipulation that "two (not necessarily peculiarly qualified) individuals cannot occupy the same substrate" is designed to rule out this possibility, and this is why *Chrysippus* would want it in the puzzle. But it is unsatisfying to simply claim, as Sedley does, that the Academics should accept it as common sense, because at this point in the argument, the Academics would have already been forced to accept that Theon and Dion have *some* of their material parts on common. And this is a strange thing to admit indeed, since Theon and Dion are not related as Siamese twins, for instance, but as part to whole. In this context – being already so far beyond the pale of common sense – it seems like a perfectly reasonable strategy for the Academics to bite the bullet and say that Dion and Theon can share *all* of their material parts, if by doing so they can forgo the additional embarrassment of admitting that Theon is dead.

The Academics' acceptance of this principle makes more sense if we consider the fact that by insisting that matter is the sole principle of identity they seem to propose a *criterion* of identity. The relevant property of a criterion is that it allows one to make *unequivocal* judgments. An underlying assumption of the

Growing Argument is that given a sufficiently precise specification of an object's material composition, one should be able to determine that object's identity unequivocally. If this were not the case, then some additional principle would be required and one could not hold that material composition is the sole principle of identity. The requirement that "two individuals cannot occupy the same substrate" seems just to reflect the view that one should assign at most one identity to any collection of matter, which follows from viewing material composition as a criterion of identity. I tried to capture this earlier by saying that the Growing Argument assumes identity to be a strict function of material composition, since when we call a relation a function, we typically mean that every element in its domain maps to *at most* one element in its codomain. Thus, the requirement that "two individuals cannot occupy the same substrate" is simply a uniqueness requirement that says that if we assign two names to the same collection of matter, they both refer to *a single individual*.

I have argued that Chrysippus' puzzle is a *reductio ad absurdum* of the Growing Argument that can be understood without any reference to "peculiarly qualified individuals." Why, then, are peculiarly qualified individuals mentioned at all if the concept seems to serve no purpose in Chrysippus' *reductio ad absurdum*? I suspect that it is because Philo is drawing from a summation of Chrysippus' attack on the Growing Argument. That summation must have included a Stoic diagnosis of why the argument fails. My conjecture is that Chrysippus began by telling the Academics that the Growing Argument fixes on an apparently commonsensical yet misguided concept of personal identity. This,

as Chrysippus saw it, is the root of all of the trouble. An entity must be "peculiarly qualified" to count as an individual because, as we have seen, attempting to define an individual solely in terms of its matter does not work. Next in the summation, Chrysippus reminds us of the general principle that he "established in advance" – that "it is impossible for two peculiarly qualified individuals to occupy the same substance jointly." Chrysippus has made it clear that the Growing Argument, so far from implying that growth and diminution are really generation and destruction, implies instead that growth results in a multiplicity of individuals that are related as parts to wholes. Moreover, even if we allow, *per impossibile* and just "for the sake of argument," that Dion and Theon are peculiarly qualified individuals sharing the same matter, it will turn out that one of them must perish, not because an individual is identical to his matter, but because of a metaphysical limitation on peculiarly qualified individuals – that they cannot occupy the same matter jointly. It may be that, according to Chrysippus, having no material parts in common is a *necessary* condition for two entities to be numerically distinct. But, at any rate, it is clear that for Chrysippus material composition cannot be a *sufficient* criterion of identity.

The irony must not have been lost on Chrysippus that he had convicted the Academics of the very absurdity they claimed was implied by the doctrine of peculiarly qualified individuals — that individuals consist of a multiplicity of entities. It is the *Academics*, rather, who have "gone crazy in their arithmetic" by taking up the Growing Argument. Of course, the peculiarly qualified individual is *not* a multiplicity but rather a single individual under different descriptions — as a

substrate, and as a peculiarly qualified substrate. This is not the way the man in the street thinks about identity, but the man in the street is often wrong. When the Stoic talks of "common conceptions" he does not mean "common opinions," and indeed, the common opinion that matter is the sole principle of identity is, on this showing, incoherent. The Academics set out to show that the very notion of personal identity is incoherent by exposing a conflict between two venerable items of common opinion – that matter is the sole principle of identity, and that identity is continuous over time. What Chrysippus' puzzle shows is that *one* of those common opinions is incoherent by itself, and this resolves the Academic *aporia*.

NOTES TO CHAPTER 2

[1]The phrase "becoming different from oneself" also sounds suspiciously Platonic, since it turns up repeatedly in several *aporiai* on change in the second part of the *Parmenides*. See *Parm*. 138c1-2, 139b5-6, 139c2-3, 139e4-4, 140a8; see also, "older/younger than oneself": 141a2-4,7, 141b1-2, 141c3-4, 141d2-3, 152d5-6, 152e2-3,9.

[2] I take the persisting, "underlying something," here, to refer to "the man" in the examples of the preceding passage. The "underlying something," in this context, is, therefore, a substance. Later in the chapter, the term "underlying something" also comes to refer to the matter that persists through a substantial change, but this represents a shift in generality. Thus, from the beginning of the chapter to 190a31, the "underlying something" refers to a substance, since it is a substance that persists through growth and alteration.

[3] David Lewis, "The Problem of Temporary Intrinsics," in *Metaphysics: The Big Questions*, eds. D. Zimmerman and P. Van Inwagen (Oxford: Basil and Blackwell, 1998), 206.

[4] Fred D. Miller, "Aristotle on the Reality of Time," *Archiv für Geschichte der Philosophie* 56 (1974): 135.

[5] Owen, "Aristotle on Time," 20. Hussey, *Aristotle's Physics III & IV,* 139.

[6] On "serious tensing" see Dean Zimmerman, "Temporary Intrinsics and Presentism," in Zimmerman and Van Inwagen, *Metaphysics*, 206-219. Another approach is to take all properties, including intrinsic properties, to be relations to times. The traditional objection to this approach is that it leaves no room for having properties *simpliciter*. I do not consider this possibility on behalf of Aristotle because I believe that the possibility of having properties *simpliciter* would be a non-negotiable desideratum of any Aristotelian theory of persistence.

[7] Sally Haslanger, "Persistence Through Time," in *The Oxford Handbook of Metaphysics*, eds. Michael J. Loux and Dean W. Zimmerman (Oxford: Oxford University Press, 2003), 323 n. 14.

[8] Dean Zimmerman, "Temporary Intrinsics and Presentism," in Zimmerman and Van Inwagen, *Metaphysics*, 212.

[9] For the way in which the past exists, see Section 2.1.3.3.3 below.

[10] Michael Frede, "Substance in Aristotle's Metaphysics" and "Individuals in Aristotle" in *Essays in Ancient Philosophy* (Minneapolis: University of Minnesota Press, 1987), 49-78.

[11] In this chapter of the *Physics*, Aristotle argues that processes must be temporally unbroken to count as numerically single, and he brings up *hexeis* like health as an analogous example. I take it that when he says "the same argument applies in each case," Aristotle is claiming that *hexeis* also must meet this requirement.

[12] Derek Parfit, *Reasons and Persons* (Oxford: Oxford Clarendon Press, 1984), Chapter 10.

[13] Philoponus and Alexander both claim that the matter cannot be replaced in one go (see Richard Sorabji, *The Philosophy of the Commentators: 200-600 AD, A Sourcebook, Volume 3, Logic and Metaphysics*, (London: Duckworth, 2005), 177). This would rule out Parfit's teletransportation scenario in advance as a case of persistence.

[14] See Alvin Plantinga, "Actualism and Possible Worlds" in *Essays in the Metaphysics of Modality* (Oxford: Oxford University Press, 2003), 103-121.

[15] Trans. Jonathan Barnes, *The Presocratic Philosophers* (London: Routledge & Kegan Paul, 1979), 106-7.

[16] I will call it the "Growing Argument," although Aristotle obviously would not have known it as such.

[17] Alexander *Mixt*. Ch. 16, 235,23 ff.; Philoponus *GC* 105,22-6, 107, 28.

[18] Sorabji, *Philosophy of the Commentators*, 176.

[19] Trans. AA. Long and D.N. Sedley, *The Hellenistic Philosophers, Volume 1, Translations of the principal sources with philosophical commentary* (Cambridge: Cambridge University Press, 1987), 171-2.

[20] David Sedley, "The Stoic Criterion of Identity," *Phronesis* 27 (1982): 255-75.

[21] By an extrinsic premise, I mean a premise that is neither explicit in the argument nor plausibly ascribed to the arguer as common sense.

[22] As parties to the previous consensus, Sedley cites M.E. Reesor, "The Stoic Concept of Quality," *American Journal of Philology* 75 (1954): 40-58; J.M. Rist, "Categories and their Uses" in *Stoic Philosophy*, ed. J.M. Rist (Cambridge: Cambridge University Press, 1969), 152-72.

[23] Sedley, "The Stoic Criterion," 269.

[24] Philo, who is on the side of the Academics, seems to just assume this principle when he speaks as if the only problem at issue after the amputation is how to determine who has died.

[25] See Sedley, "The Stoic Criterion," 272 n. 17.

[26] At *Comm. not.* 1059B ff., Plutarch lays out the dialectical context for the dialogue in which the account of the Growing Argument is given (at 1083A-1084A): The interlocutor of Diadumenus has just come from a group of Stoic friends who have been denouncing the "older Academics." The interlocutor says that one of his friends had opined that it was providential that Chrysippus had come after Arcesilaus and before Carneades, because by means of his rejoinders to Arcesilaus, Chrysippus had left many aids to sense perception. Given this

background, I follow Sedley in ascribing the Academic formulation of the Growing Argument given at 1083A-1084A to Arcesilaus. (See Sedley, "The Stoic Criterion," 270.)

[27] See above, p. 126-7, for reasons why this premise must be part of the puzzle. Sorabji on p. 184-5 of his *Philosophy of the Commentators*, provides an interpretation that does not include this premise, but I do not see how he can justify its exclusion from the puzzle, given the reasons in my footnote 61.

[28] Sedley, "The Stoic Criterion," 270.

[29] One may cavil that what follows is too bizarre, and that if Chrysippus meant something like this, he would have had Theon grow a mole instead of a foot. I will show in the sequel, however, that bringing in the growth of a discrete new part makes better overall sense of the puzzle, even though it produces a scenario that is biologically impossible.

[30] Trans. Barnes, *Presocratic Philosophers*, 106-7.

[31] Trans. Long and Sedley, *Hellenistic Philosophers*, 166.

[32] Note that this is different from saying that the "number or quantity" of material parts alone is criterial for identity, which would yield the absurd consequence that all equally numbered sets are identical.

[33] If one is inclined to insist that discreteness is an ineliminable part of the concept of individuality, and that it may be assumed as common sense in any discussion of individuality, then, of course, my solution will not be attractive. For my part, I see no reason to regard the claim that individuals are discrete as an analytic truth.

Chapter 3: Limitations of the Aristotelian Concept of Motion

1.0 INTRODUCTION

We have seen that a central feature of Aristotle's concept of motion is the idea that a moving object passes between opposing termini. The termini of a motion serve a number of important functions. They supply key identity conditions of a motion, and in the case of the *terminus ad quem*, they also provide a normative goal, which I have shown can be used to define the order of the motion's phases. We also saw that the termini are related to each other by a special teleological potentiality, which provides the metaphysical basis for this ordering, and which may also be used as the basis for establishing the diachronic identity of the moving object. These are substantial philosophical benefits, and do much to commend Aristotle's general approach. But the approach, I think, is not without disadvantages, and I will highlight one of these disadvantages in the present chapter, viz., the idea that motion is simply the traversal from place A at time t to place B at time $t + 1$ makes it impossible for Aristotle to account for how motions come to be and pass away. For this purpose, I will consider an ancient puzzle about motion that first appears in Plato at *Parmenides* 155e-157b.

1.1 The puzzle

The puzzle, as Plato gives it, can be roughly summarized as follows:

> At every time, a given object must either be in motion or at rest; there is no third possibility. Also, an object can never be simultaneously both in motion and at rest. The only way for an object to be both in motion and at rest is for it to be in motion and at rest at different times. But how does a thing come to be in motion at one time and at rest at another? It cannot switch at a time when it is in motion. Nor can it switch at a time when it is at rest. This would seem to exhaust the possibilities for the times when the switch could occur. But a thing can not change without changing.

Plato asks, essentially, when and how an object switches between motion and rest. Aristotle takes up the puzzle in the sixth book of the *Physics (Phys.* 234a34-b5), but he recasts it as a problem about what to call an instant separating a period of motion from a period of rest; i.e., shall we call it an instant of motion, an instant of rest, an instant of both motion and rest, or an instant of neither motion nor rest? Saying that the dividing instant is an instant of both motion and rest violates the law of non-contradiction, while saying that it is an instant of neither motion nor rest violates the law of the excluded middle. But saying that it is just an instant of motion or just an instant of rest seems arbitrary, since it bounds both the period of motion and the period of rest.

Plato's and Aristotle's versions of the puzzle each focus on different but important aspects of the transition between motion and rest. Aristotle inquires about the status of the instant dividing periods of motion and rest, while Plato inquires about what is involved in arriving at and departing from this instant. Plato supposes that the transition between motion and rest involves an instantaneous event of switching, which has the disadvantage of exacerbating the problem of what to call the state in which it occurs; i.e., since the switch is itself a motion, how can the switch be a switch between motion and rest? Since

Aristotle's formulation of the puzzle is unencumbered by this mistake, and it is surely a mistake, I will concentrate on it, but still honor Plato's point that it is essential to give an account of what is involved in arriving at and leaving the instant of change. As for solutions to the puzzle, I will consider both Aristotle's solution, and what I think is the best solution from the modern literature, which is the one proposed by Richard Sorabji in his *Time, Creation and the Continuum*.[1]

1.2 Aristotle's solution

Aristotle's exposition of the puzzle is somewhat truncated, compared to the one I have given above and the one that appears in Sorabji. Aristotle does not consider the possibilities of saying that the instant in question is just an instant of motion or just an instant of rest (the latter option is the one advocated by Sorabji). Rather, he assumes that, *if* one allows motion or rest at an instant, then one must say that an instant dividing periods of motion and rest must be an instant of *both* motion and rest, since "it is the same instant that belongs to both the periods [of motion and of rest]" (*Phys*. 234a34). Aristotle argues that, since, the law of non-contradiction is non-negotiable, we must reject the assumption that there can be motion or rest at an instant, and this, he implies, avoids violating the law of the excluded middle because "the motion of that which is in motion and the rest of that which is at rest must occupy [a period of] time" (*Phys*. 234b8-9). Aristotle's point seems to be that the law of the excluded middle does not apply to motion and rest at instants because they are not the sort of things to exist at instants — motion and rest are defined over periods of time. This allows Aristotle to solve

the puzzle by saying that the instant dividing periods of motion and rest is an instant of neither motion nor rest, and that this is the case because *all* instants are instants of neither motion nor rest.

1.3 Sorabji's solution

Sorabji proposes to solve the puzzle by giving an argument why the instant dividing a period of motion and a period of rest should be an instant of rest, or, what amounts to the same thing, why there should be a first and a last instant of rest, but no first or last instant of motion. His key assumption is that deceleration to and acceleration from a stand be continuous, so that there is no first or last velocity above zero. If we also make the natural assumption that something is in motion if and only if it has a positive velocity, then there will be no first or last instant of motion, and this at least poses no problem with supposing that there is a first and last instant of rest; i.e., an instant of rest at the beginning of a motion which is the last instant of a previous period of rest and an instant of rest at the end of a motion which is the first instant of a succeeding period of rest, where each instant of rest is an instant of zero velocity.

1.4 Problems with Aristotle's solution

A difficulty with Aristotle's solution is that it creates a mismatch between the spatial and kinetic termini of a motion, because the spatial termini can be punctual while the kinetic termini cannot. By spatial termini, I mean the places at which a motion begins and ends. By kinetic termini, I mean the states of rest which the

moving body passes from and to at the beginning and end of every motion. According to Aristotle, spatial termini can be punctual while kinetic termini cannot because, while it is possible to be in a punctual place at a punctual time, it is not possible to be in a state of rest at a punctual time. A moving object is not at rest, according to Aristotle, until some period of time has elapsed at the same location (*Phys*. 234a24-b9; 237a14-15; 263b21). In Aristotle's account, unlike Sorabji's, there is no last instant of rest to bound a motion at the beginning, nor is there a first instant of rest to bound it at the end (*Phys*. 234a24-b9; 237a14-15). The instant bounding a period of motion and rest does not belong to either the period of motion or the period of rest, because it is an instant of neither motion nor rest. The difficulty with this view is that there will be an instant at the end of every motion in which the spatial terminus has been reached, while the kinetic terminus has not, because a period of time will need to have elapsed for the body to be at rest.

Sorabji's solution to the puzzle solves this problem by envisaging a last instant of rest to bound a motion at the beginning and a first instant of rest to bound it at the end. Thus, he can say that, at the end of a motion, the moving body will reach the kinetic and spatial termini at the same time.[2] But there is another, and perhaps more important point at which Sorabji's solution succeeds where Aristotle's does not. The original puzzle, as Plato conceived it, wondered not only about the status of the instant dividing periods of motion and rest, but also about what is involved in arriving at and leaving this instant. Sorabji's solution solves this problem beautifully by supposing that there is no first or last

velocity above zero, and that the velocity increases continuously from a value of zero at the beginning of a motion and decreases continuously to a value of zero at the end of a motion.

1.5 What about having no first or last *non-instantaneous* velocity above zero?

Sorabji's solution obviously differs from Aristotle's in that it envisages rest at an instant. It is also clear that it envisages motion at an instant, insofar as it makes an instant of rest a limiting case of instantaneous velocity (i.e., where velocity is equal to zero). There is a way, however, to get Sorabji's condition that there is no first or last velocity above zero *without* supposing that there is motion at an instant. In the case of coming to a stand at the end of a motion we need only suppose that there is an infinite sequence of motion subsegments with *non-instantaneous* velocities that converge or pass to zero, e.g., with the times and distances divided into subsegments according to an infinite geometric sequence such as 1/2, 1/4, 1/8, …, $1/2^n$, … (n = 1, 2, 3, …). Similarly, in the case of accelerating from a stand at the beginning of a motion we need only suppose that there is an infinite sequence of motion subsegments with *non-instantaneous* velocities that pass from zero, e.g., with the times and distances divided into subsegments according to an infinite geometric sequence such as …, $1/2^n$, …1/8, 1/4, 1/2 (n = … 3, 2, 1). The condition that there is no first or last non-instantaneous velocity above zero will hold if it is possible for the moving object to pass through all of the motion subsegments in the course of passing to or from the instant bounding the end or beginning of the motion. This solves Plato's

problem of explaining what is involved in arriving at and departing from the instant of change. It also addresses Aristotle's problem of reaching the spatial terminus before reaching the kinetic terminus. Following this strategy, one can use the instant at which zero is reached —an instant that coincides with the arrival at the spatial terminus— as the kinetic terminus. And one can do this even if the instant is one of neither motion nor rest. Thus, for instance, Aristotle could say that as a train reaches its destination, it passes through an infinite decreasing sequence of *non-instantaneous* velocities — i.e., distances traveled over times elapsed — and that *this* is what it means to reach the kinetic terminus.

2.0 AN ARISTOTELIAN ARGUMENT THAT THERE IS NO FIRST OR LAST VELOCITY ABOVE ZERO?

At first sight, there appears to be an account of acceleration and deceleration in *Physics* book 6 that fits this description. Indeed, Aristotle's argument that there is no first time of coming to a stand (*Phys*. 238b23-239a10), since it assumes that times of coming to a stand are infinitely divisible into subintervals that are also times of coming to a stand, appears to enable us to construct a scenario from Aristotelian premises where there is no first or last non-instantaneous velocity above zero.

The argument in question can be summarized as follows: Since there is no motion at an instant (*Phys*. 234a24-b9; 237a14-15), times of motion must be periods of time (*Phys*. 239a3-b4). Since times of motion are periods of time and periods of time are infinitely divisible, times of motion must also be infinitely divisible. Since times of coming to a stand are times of motion, times of coming

to a stand must be infinitely divisible as well. If times of coming to a stand are infinitely divisible, there is no first time of coming to a stand.

To explain why this is so, Aristotle imagines trying to single out a first period of coming to a stand. Since this period can be divided in half, the first half of the period that was singled out is earlier than the undivided period taken as a whole. Likewise, this half period can be divided again, and the first quarter period will be earlier than the undivided half period taken as a whole. Thus, the period initially taken to be first may be divided according to the infinite geometric sequence …, $1/2^n$, …1/8, 1/4, 1/2 (n = … 3, 2, 1), where each of an infinite number of non-overlapping subintervals of time is a better candidate for being the first time of coming to a stand than its successor (*Phys*. 238b23-239b4). Aristotle should also say that there is no last time of coming to a stand for the same reasons, since any candidate for the last period of coming to a stand may be divided according to the infinite geometric sequence 1/2, 1/4, 1/8, …, $1/2^n$, … (n = 1, 2, 3, …), where each of an infinite number of non-overlapping subintervals of time is a better candidate for being the last time of coming to a stand than its predecessor.

This argument rests on what is probably the unwarranted assumption that times of coming to a stand are infinitely divisible into subintervals that are also times of coming to a stand. This assumption is unwarranted because a thing might momentarily halt its deceleration within a time that still meets the minimal condition for being a period of coming to a stand, viz., any time period t which is a period of coming to a stand must be divisible into at least two non-overlapping

subintervals such that for all n, if $n > 1$, then $d_n/t_n < d_{n-1}/t_{n-1}$, where d_n is the distance traveled during subinterval n, and t_n is the duration of subinterval n. But for Aristotle to argue that there is no first or last time of coming to a stand based on the infinite divisibility of time and distance, a much stronger condition is required, viz., times of coming to a stand must be divisible into *an infinite number* of non-overlapping subintervals such that for all n, if $n > 1$, then $d_n/t_n < d_{n-1}/t_{n-1}$. Thus, for any time period t of coming to a stand, to any infinite sequence of non-overlapping subintervals of this period (e.g., with their lengths determined by the sequence 1/2, 1/4, 1/8, …, $1/2^n$, … (n = 1, 2, 3, …)) there must correspond an infinite sequence of non-overlapping subsegments of distance traversed which result in an infinite decreasing sequence of non-instantaneous velocities $v_n = d_n/t_n$ for each of the non-overlapping subintervals of time and subsegments of distance.

On this account, a period of coming to a stand in which the end coincides with the end of the motion as a whole will be a period in which an infinite decreasing sequence of non-instantaneous velocities converges or passes to 0 as n becomes infinitely large. So it is possible to give an account of deceleration to a stand where there is no last non-instantaneous velocity above zero using a common sense definition of what it means to come to a stand combined with Aristotle's assumption that times of coming to a stand are infinitely divisible into subintervals that are also times of coming to a stand.

This is an interpretation of an isolated passage, of course, and the plausibility of ascribing it to Aristotle depends on whether it can be squared with the rest of Aristotle's philosophy. I am not optimistic that this can be done,

particularly in the light of certain aspects of Aristotle's philosophy of mathematics. By saying that an infinite sequence of non-instantaneous velocities converges or passes to 0 as *n* becomes infinitely large, we have attributed to Aristotle the idea that one can traverse the whole of an infinite sequence. There is good reason to doubt, however, that Aristotle believed that this could be accomplished. And I will argue that Aristotle must accept that the whole of this sequence must be traversed, otherwise, he cannot claim that that the moving body ultimately arrives at a state of rest. If the whole of the infinite sequence is not traversed, one is entitled to ask when and how, precisely, does the transition from motion to a period of rest take place?

3.0 ARISTOTLE AND INFINITE SEQUENCES AND SERIES

Aristotle discusses infinite sequences and series in connection with two different sorts of applications: geometrical applications having to do with the exhaustion of magnitudes by means of inscription or partial summation, and kinematic applications relating to the Zenonian paradoxes of motion. An example of a geometrical application is found in book 3, chapter 6 of the *Physics*:

> If we take a determinate part of a finite magnitude and add another part determined by the same ratio (not taking in the same amount of the original whole), and so on, we shall not traverse the given magnitude. But if we increase the ratio of the part, so as always to take in the same amount, we shall traverse the magnitude, for every finite magnitude is exhausted by means of any determinate quantity however small. (*Phys*. 3.6 206b6-12)

"A determinate part of a finite magnitude" is just some fraction of a finite magnitude; e.g., one half. So to take "another part determined by the same ratio"

just means to take, for instance, half (i.e., the same ratio) of what is left, e.g., one quarter. "Not taking in the same amount of the original whole" means, e.g., taking one quarter in the second step instead of taking one half again. "And so on" means that we extend this process *ad infinitum* (i.e., 1/2 + 1/4 + 1/8 + …) so that the sum will approach, *ad infinitum*, the "finite magnitude" which I have assumed to be 1 unit in size for illustration. Aristotle would have been familiar with this notion of infinite approach from the geometry of his day, and in particular, from the problem of squaring the circle. Aristotle showed a keen interest in the problem of quadrature, mentioning it in six passages by my count,[3] three of which addressing proofs of specific geometers (Bryson, Antiphon, and Hippocrates). These proofs undertook to "exhaust" the area of a circle by inscribing, one after another, successively larger polygons within it, a strategy that relies on a type of infinite approach very similar to what we just saw in Aristotle, where the areas of successively larger inscribed polygons were taken to tend, *ad infinitum*, toward the area of a circle.

An example of a kinematic application of infinite sequences and series is found in Aristotle's discussion of Zeno's dichotomy paradox. The paradox is addressed in four places: *Topics* 160b7-9; *Physics* 233a21-31; *Physics* 239b9-14; *Physics* 263a4-b8.[4] At *Physics* 239b9-14, we are told that Zeno argued for "the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal." "The non-existence of motion" is glossed as "it is impossible to move or to traverse the stadium" at *Topics* 160b8-9 and "it is impossible for a thing to pass over or severally to come

in contact with infinite things in a finite time" at *Physics* 233a22-3. This is usually[5] taken to mean that the paradox envisages a runner running from point A to point B in some finite time, but that the run is thought to consist of an infinite number of sub-runs in which the lengths of the times or distances traversed are determined by the infinite geometric sequence …, $1/2^n$, …1/8, 1/4, 1/2 (n = … 3, 2, 1). On the assumption that it would take an infinite amount of time to complete an infinite number of sub-runs, Zeno draws the paradoxical conclusion that the runner can never finish the race. At *Physics* 233a21-31, Aristotle gives the answer that we would expect, viz., that traversing the distance does not take an infinite amount of time because running across ever- reduced distances takes ever-reduced amounts of time. But then at *Physics* 263a4-b8, he makes the further claim that the ultimate, and philosophically correct answer to the puzzle, is that the runner can finish the race because there are only a finite number of segments in the race course in actuality, even though there may be as large a finite number as one likes.

This seems like an odd and unnecessary move unless we take into account an assumption behind both the geometrical and kinematic applications of infinite series just outlined. In each of these cases, Aristotle is working with the assumption that passing through an infinite series is a concrete step-wise process, and is, therefore, indefinitely extendable, and therefore always incomplete. Thus, the infinite approach in the geometrical application is not only infinite in that it involves an infinite series, but in that it would require an infinite amount of time to complete. This is what Aristotle means when he says, at *Physics* 3.6 206b6-12,

that "we shall not traverse the given magnitude." And in the case of the kinematic application, the implication of *Physics* 263a4-b8 is that if the runner *were* actually required to pass through an infinite sequence of half-distances, he would indeed, never have finished the race. What saves him from this fate is not a way of passing through the infinite sequence that is not concrete and step-wise, but a way to avoid going through the sequence altogether.

3.1 Constraints on geometrical concepts

But why should anyone think that passage through an infinite series or sequence must be accomplished in a step-wise manner? To see Aristotle's motivation for this in the geometrical application, we will need to understand Aristotle's concepts of geometrical objects and geometrical reasoning. For Aristotle, geometrical objects are just concrete physical objects under a certain description. In Aristotle's view, when a geometer talks about circles, he is actually talking about concrete things like shields; he is talking about shields *qua* circular, just not about circles *qua* properties of shields. Put another way, geometers focus on one aspect of shields — their circularity — (or at least this is what Aristotle thinks that they should do if they see things aright) but they do not suppose that circles exist apart from the shields of which they are properties (*Phys*. 193b31-33). As for geometrical reasoning, Aristotle tells us in *Metaphysics* book Θ chapter 9 1051a21-31 that geometrical facts are discovered by literally constructing geometrical figures (e.g., performing operations like dividing a magnitude) in one's mind.[6] Thus, the character of geometrical figures, such as infinitely

segmented geometrical magnitudes, is constrained by the process by which they are imagined to have come about. And this process, while it need not involve the physical manipulation of magnitudes and shapes, nevertheless depends on our intuitions about these physical sorts of operations.

3.2 Ancient geometers and infinite series

Although geometers in antiquity were probably not motivated or even exercised by philosophical concerns of this nature, they also conceived of passage through an infinite sequence as a concrete step-wise process. Infinite series were a central feature of the "method of exhaustion," which was used in attempts to square the circle, among other things, from the 5th century onward. In the many instances where this method was employed, exhaustion was invariably conceived of as the *process* of inscribing, one after another, successively larger rectilinear polygons within a curvilinear figure. For example, in the proof that is generally attributed to Eudoxus, appearing in book 12, proposition 2 of Euclid's *Elements*, we find the claim that we can exhaust any proper sub-area of a circle "by bisecting the remaining circumferences and joining straight lines, and by doing this continually" (*Elements* 12.2.33-37).[7] The justification for this claim is the principle that

> if unequal magnitudes be set out, and if from the greater there be subtracted a magnitude greater than the half, and from that which is left a magnitude greater than the half, and if this process be done continually (καὶ τοῦτο ἀεὶ γίγνηται), there will be left some magnitude which will be less than the lesser magnitude set out. (*Elements* 12.2.37-41)[8].

This principle is simply the "converse" of what was later to be called the axiom of Archimedes,[9] but which we have just seen Aristotle express at *Physics* 3.6 206b6 as

> But if we increase the ratio of the part, so as always to take in the same amount, we shall traverse the magnitude, for every finite magnitude is exhausted by means of any determinate quantity however small. (*Phys*. 3.6 206b6)

3.3 The impossibility of squaring the circle

Aristotle seemed to think that squaring the circle was impossible (*EE* 2.10 1226a28-30) or at least not yet accomplished (*Cat*. 7 7b27-33), and that allegedly successful quadratures either contained various fallacies (*Soph. Ref*. 11 171b13-18; *Post. An*. 1.9 75b37-76a3) or were based on erroneous first principles (*Phys*. 1.2 185a14-17). His attitude seems to be that if one takes into account the principle that magnitude is infinitely divisible, then one should realize that the indefinitely extendable step-wise process of inscription can never be completed. In book 1 of the *Physics*, for instance, Aristotle mentions Antiphon's attempted proof:

> Moreover, no man of science is bound to solve every kind of difficulty that may be raised, but only as many as are drawn falsely from the principles of the science: it is not our business to refute those that do not arise in this way: just as it is the duty of the geometer to refute the squaring of the circle by means of segments, but it is not his duty to refute Antiphon's proof. (*Phys*. 1.2 185a14-17)

The ancient commentators interpret Aristotle to mean that Antiphon's proof of the squaring of the circle is based on false first principles, rather than on

a fallacious deduction from true first principles. Simplicius describes Antiphon's quadrature as directly attempting to exhaust the area of a circle by means of successively doubling the number of sides of an inscribed polygon: "and doing this always, so that at some time, the area being exhausted, a certain polygon would be inscribed in this manner in the circle whose sides on account of smallness would coincide with the arc of the circle." (*In phys*. 9.55.4-8)[10] Eudemus (*apud* Simplicium), thought that the false first principle at issue involved the denial of the infinite divisibility of magnitude, and that given the (in his view, correct) Aristotelian assumption that magnitude is infinitely divisible, Antiphon's quadrature will never result in the exhaustion of the area of the circle.

3.4 Finitism in ancient geometry

Aristotle is right, of course, since exhaustion *is* impossible, as long as it is conceived of as *direct* exhaustion, viz., as the actual, step-wise inscription of an infinite sequence of regular polygons within a circle. Eudoxus was later able to prove that continually inscribing successive regular polygons within a circle will leave segments of the circle which will be less than any given area. But this does not amount to the claim that one can exhaust the area of a circle by actually inscribing an infinite sequence of polygons in it. Without knowledge of this use of exhaustion, one can see how Antiphon's proof could have become emblematic, for Aristotle, of its impossibility, as well as of the impossibility, in general, of passage through an infinite sequence by means of an indefinitely extendable step-wise process.

But even if Aristotle had known of the proofs of Eudoxus and, anachronistically, of those of Archimedes, they would have given him no reason to change his mind about the impossibility of passing through the whole of an infinite sequence. For the conception of exhaustion is just as discrete and process-bound in Eudoxus as it is in Antiphon. In fact, as Knorr points out, the description of the process of exhaustion in *Elements* 12.2 — "cutting the remaining circumferences in two and joining straight lines and doing this continually" — is almost a verbal echo of the description that Simplicius gives of Antiphon's quadrature — "cutting the sides of the 16 sided figure, and joining the straight lines and doubling the inscribed polygon and doing this continually."[11]

Moreover, the proofs of Eudoxus and Archimedes took care to avoid traversals of infinite series by stating their infinite series in finite forms. As Knorr points out, "Archimedes, in *Quadrature of the Parabola*, prop. 23, established in finite form the equivalent of the summation $1 + 1/4 + 1/4^2 + 1/4^3 + \ldots = 4/3$." The "finite form" that Knorr refers to is the theorem that for any finite $n \geq 1$, the area A bounded by a straight line and a section of a right-angled cone is equal to the series $A + A/4 + A/16 + A/64 + \ldots + A/4^n + 1/3(A/4^n) = 4A/3$. Note that instead of ending with "…" Archimedes' series ends with the plug "$1/3(A/4^n)$" that eliminates the discrepancy between the nth partial sum and the area of the conic section. Archimedes' idea here is that when one leaves off summing the areas of inscribed polygons, as one inevitably must, one must always round up the discrepancy between the nth partial sum and the area of the conic section. By contrast, the sense of putting "…" at the end of a series is that as n becomes

infinitely large, this discrepancy may be neglected. But since the step-wise process of inscription will never literally exhaust the area under the parabola, Archimedes does not feel entitled to talk about a case where an infinite number of inscriptions have been completed.

3.5 Verb tense and infinite series

This same attitude can also be seen in Aristotle, in his resistance to admitting that there is any genuine sense in which the runner in Zeno's dichotomy paradox can be said to have completed an infinite number of runs.[12] From this, I believe that one can make a general point about verb tense in describing passage through an infinite sequence: In Aristotle's opinion, one can say that something *is passing* through an infinite sequence, but one can never say that something *has passed* through an infinite sequence. Nor can one say that something will pass through an infinite sequence in an infinite amount of time, even though from a certain modern perspective, a perfectly intelligible sense may be given to such a claim. Thus, Knorr complains of Aristotle's argument in *Physics* 6.7 for the claim that it is impossible for a thing to undergo a finite motion in an infinite time: "Having admitted nonuniform motion, we might easily construct a decelerating motion which requires some fixed time to traverse each successively diminishing proportional part of the distance; then the time to traverse the whole will indeed be infinite." What Knorr misses, I think, is that since passage through an infinite sequence is conceived of in terms of an indefinitely extendable step-wise process,

it is essentially *incomplete*, and to say that it will be complete in an infinite amount of time would have struck Aristotle as simply incoherent.

4.0 ARISTOTELIAN INFINITY

In Section 1.6, I suggested that Plato's and Aristotle's puzzle about stopping and starting might be solved by supposing that there is no first or last *non-instantaneous* velocity above zero, and that, in the case of deceleration to a stand, for instance, the moving body traverses an infinite sequence of non-instantaneous velocities that converges or passes to 0. And indeed, in Section 2.0, we saw that Aristotle's argument that there is no first time of coming to a stand seemed to suggest that this solution would have met with his approval.

But we just saw that Aristotle thinks of an infinite sequence as something that we can never pass through in its entirety, and this clearly conflicts with any solution that supposes there is no first or last non-instantaneous velocity above zero. I have suggested a reason why Aristotle takes this view, viz., he allowed, as did every geometer of his day, his notion of passage through an infinite sequence to be constrained by intuitions about the manipulation of physical magnitudes and shapes. If this constraint were unconscious, as I think it was for Aristotle, it is easy to see how passage through an infinite sequence would seem unavoidably, and even axiomatically incomplete.

This goes a long way toward explaining why Aristotle could not have thought passage through an infinite sequence to be a viable option for explaining the kinematics of starting and stopping. But there is, perhaps a deeper reason why

Aristotle thinks of infinite sequences in this way, and it has to do with his concept of infinity. It is almost an analytic truth for Aristotle, that the infinite, properly understood, is untraversable. "What we are interested in," he says, "is 'infinite' in the sense of 'untraversable.'" (*Phys*. 204a14) And in fact, when he surveys the possible senses of "infinite" at *Physics* 204a2 ff., his chief concern is *which* sense of untraversable to settle upon, not whether or not the infinite is traversable.

Why is the infinite so obviously untraversable to Aristotle? Again, in the light of what I have said above, one might say that it is because he thinks of traversal in terms of discrete, step-wise processes, like dividing and counting. This is certainly part of it, but even if Aristotle had been working with another concept of traversal, it is unlikely that he would have changed his mind, since, in a sense, he did not even think that the infinite is there to be traversed. What I mean is that Aristotle does not think that an infinite sequence pre-exists an attempt to traverse it, and even while such an attempt is in progress, the sequence is never more than a finite number of stages long. This might seem like just an elaborate way to say that there are no infinite sequences, but the motivation behind it is clear. In *Physics* book 3.4-8, Aristotle wants to reserve some sense of "infinite" to ensure that magnitudes are divisible *ad infinitum*, that time has no limit, and to accommodate the fact that the natural number series does not give out in thought. In other words, he needs a sense of infinite to allow that some processes can (like counting or dividing a magnitude), and other processes do (like the advance of time) go on and on indefinitely. But our present problem is not why processes can go on and on but how they manage to start and stop, and it

is not surprising that a solution devised for the former problem will be unsuitable for the latter.

4.1 The traditional interpretation

The sense of infinity in question is what Aristotle calls "potential infinity." The traditional interpretation of this concept emphasizes a passage where, after concluding that the infinite has a potential existence, Aristotle says:

> But the phrase "potential existence" is ambiguous. When we speak of the potential existence of a statue we mean that there will be an actual statue. It is not so with the infinite. There will not be an actual infinite. The word "is" has many senses, and we say that the infinite "is" in the sense in which we say "it is day" or "it is the games," because one thing after another is always coming into existence. For of these things too the distinction between potential and actual existence holds. We say that there are Olympic games, both in the sense that they may occur and that they are actually occurring. (*Phys*. 206a18-25)

Simplicius takes Aristotle to be distinguishing, here, between senses of actuality and potentiality that attach to two distinct modes of existing, viz., existing "as a whole … like a man or a house" on the one hand, and existing "as what has its being in becoming, like a contest and a day" on the other (*In phys*. 492, 23-5). The distinction is roughly the same as the one that endurantists make between "continuants" and "occurrents" respectively, where "continuants" are three-dimensional objects like men or houses that have only spatial parts, and wholly exist at each moment of their existence, and "occurrents" are four-dimensional events or processes like days or games that have temporal parts, and only partly exist at each moment of their existence. The senses of potentiality and

actuality that Aristotle warns us not to associate with potential infinity are the senses associated with continuants like statues and pieces of bronze. The reason for this warning is that a piece of bronze is a potential statue, which can become an actual statue if it is molded into a certain shape. Something that is potentially infinite, however, can never become actually infinite. The senses of potentiality and actuality that are appropriate to the infinite, rather, are the senses associated with occurrents like days and Olympic games. Days and Olympic games are actual while they are occurring, but they are also always potential in the sense that some of their temporal parts are always in a time other than the present. Likewise, all but a finite number of the infinite succession of past days and generations of men are in the past and have ceased to be, and this allows Aristotle to account for their infinity by saying that their number is potentially, but not actually infinite. What this means, Simplicius implies, is that we may have infinitely many successive things, but not infinitely many simultaneous things.

4.2 The problem of infinite precession[13]

This is the main thesis of Simplicius' interpretation, and the text seems to confirm it, since at the end of book 3 of the *Physics*, Aristotle claims that time, change and thought avoid being actual infinities *because* their parts do not persist (*Phys*. 208a5-6; 20-1). But if this is Aristotle's view, he is guilty of a serious mistake; the mistake first pointed out by Philoponus in the 6th century A.D. Philoponus, who as a Christian, was trying to disprove the Aristotelian doctrine that time and the world had no beginning, argued that if this were the case, then the sun would

have already gone around the earth an infinite number of times. Aristotle, however, stresses that the infinite is untraversable (*Phys*. 204a14), and that it is impossible, for instance, to divide a magnitude an infinite number of times or to count to infinity; impossible, that is, not because no one has the requisite physical or mental capacity to carry out these tasks, but "because it is not in the nature of the infinite to be traversed (the sense in which the voice is 'invisible')" (*Phys*. 204a3-4). In other words, the infinite cannot be traversed because to do so would be inconceivable. But it is hard to see how the sun going around the earth an infinite number of times is any less a traversal of an infinite sequence than the making of an infinite number of divisions or counting to infinity. What this attack made clear is that while the failure of past years to persist may exempt them from being included in a set of simultaneous existents, their succession nonetheless implies the traversal of an infinite sequence, at least if Aristotle wishes to maintain that time and the world had no beginning.[14]

4.3 An *ad hoc* response

In the light of this problem, if all that the distinction between the potential and the actual in Aristotle's account of infinity amounted to was a distinction between the successive and the simultaneous, it would be a disappointing theory. As it happens, however, I believe this doctrine (that the failure of times and generations of men to persist solves the problem of infinite precession) is a mere appendage, developed as an *ad hoc* response to an objection much like the one raised by Philoponus. The only clear endorsement of it comes in the passage at the end of

book 3 where Aristotle takes himself to be disposing of objections to his theory. In the light of this, my guess is that Aristotle first formulated his theory of the potential infinite with a view to accounting for indefinitely long succession, and then in response to an objection involving indefinitely long precession (perhaps from someone else, or perhaps from himself), he retrofitted his theory to address this problem. I concede that it was a disappointing and inadequate retrofit to a very serious and difficult problem, but I wish to show, contrary to what the traditional interpretation would seem to suggest, that there is more to the theory than this.

4.4 Infinite succession

As I argued earlier, Aristotle was mainly interested in finding a sense of infinity to accommodate the fact that some processes can, and other processes do go on and on indefinitely. He speaks of "dividing *ad infinitum*" (*Phys*. 206b5-6), where the "*ad*" translates εἰς, which with the accusative implies a movement toward but not an arrival at infinity. And when he elaborates on the sense in which the infinite exists as occurrents do, like days and Olympic games, he describes this existence in terms of succession, not precession: "one thing *after* another (ἄλλο καὶ ἄλλο) is always coming into existence" (*Phys*. 206a22); "one thing is always being taken *after* another (ἄλλο καὶ ἄλλο)" (*Phys*. 206a27-8). The examples of infinities Aristotle is trying to account for are disparate (e.g., time, generations of men, divisions of a magnitude), and, indeed, major differences exist between them: in spatial magnitudes, what is taken persists; in time and the generations of

men it does not. But Aristotle emphasizes that, even so, they all exist as "one thing is always being taken *after* (ἄλλο καὶ ἄλλο) another" (*Phys*. 206a27-8).

4.5 The future-bias of potential

This focus on succession over precession in Aristotle's exposition of potential infinity reflects a future-bias in the concept of potentiality. "No potential relates to being in the past," Aristotle tells us, "but always to being in the present or future." (*Cael*. 1.12 283b13-14) This is because, in Aristotle's view, though what is past is necessary (*EN* 1139b7-9), necessity does not imply possibility. Rather, Aristotle's notion of possibility, as he defines it in *De interpretatione*, is of a mode of neither impossibility nor necessity; that is to say, it is a mode of contingency. Now one might think that this is all the worse for Aristotle, since, the infinite past sequence of celestial rotations is on this account not only actual, but necessary. And one would be right to think this, but my point is that, given the future-bias of the notion of potentiality, the concept of potential infinity could not have been originally intended to solve Philoponus' problem of precession.

4.6 Existing potentially

Indeed, sentences like "one thing *after* another (ἄλλο καὶ ἄλλο) is always coming into existence" (*Phys*. 206a22) suggest the existence of an inexhaustible store of unfulfilled *future* possibilities for dividing, counting, and etc., and it appears that this is what it means for the infinite to exist δυνάμει (*Phys*. 206b13). "The infinite" denotes what is infinite or what has the property infinity, viz.,

processes of dividing, counting, and etc. For the infinite to exist δυνάμει is for processes to have these unfulfilled potentialities. Charlton objects that this cannot be so.[15] If all there is to existing δυνάμει is to have unfulfilled potentialities, "we should all exist δυνάμει." But since we are told that infinity exists as processes exist, this objection ignores a relevant difference in the way in which substances and processes have potentialities. Aristotle tells us at the beginning of book 3 of the *Physics* that a motion only exists insofar as, and as long as, it has the unfulfilled potentiality of being completed by the arrival of the moving thing at a goal state that is intrinsic to the motion. A motion is an actuality of a potentiality for a moving thing to be *en route* to a goal, but as long as the moving thing is *en route*, the motion is an actuality *qua* existing potentially since it is potentially, but not actually completed.[16] Once this potentiality is realized, the motion no longer exists, but as long as it does exist, the motion has this unfulfilled potentiality. So one could say that a motion always has an unfulfilled potentiality, but this is different from saying that someone always has the potential to be a concert violinist. Substances, of course, always have unfulfilled potentialities, including potentialities to achieve states that are uniquely determined by their natures, but they do not exist insofar as, and as long as they have these potentialities, as changes do.

4.7 Existing incompletely

Moreover, substances are not incomplete by virtue of having unrealized potentialities, whereas processes are. "Change," says Aristotle, "is thought to be a

sort of actuality, but incomplete, the reason for this view being that the potential whose actuality it is is incomplete" (*Phys*. 201b31-33). Aristotle refers back to this conclusion a number of times, at *Physics* 257b6-9, *De anima* 417a16-17, *De anima* 431a6-7, and *Metaphysics* 1048b29-30, but at *Nicomachean Ethics* 1174b2-5 he gives the reason for this incompleteness, viz., "the whence and whither give [changes] their form." Change is not simply the actuality of a potentiality to be in any state, but rather, it is the actuality of a potentiality to be in a state that is the incomplete realization of a particular goal (i.e., the "whither"). If manhood is the goal, for instance, change is the actuality of the potentiality to be *en route* to manhood. It is the actuality of the potentiality to be a teenager, for instance. But while the process of growth may be said to be incomplete, the teenager himself is not. He has the same form or species essence as the grown man, and none of his constitutive parts are missing. A thing is incomplete if it has some of its constitutive parts missing, and indeed some of the temporal parts of a process are always absent.

Commentators have often assumed that this is what Aristotle means when he says that the potential infinite exists as processes do and then describes the potential infinite as incomplete, as he does in the following passage:

> A quantity is infinite if it is such that we can always take a part outside what has been already taken. On the other hand, what has nothing outside it is complete and whole. For thus we define the whole—that from which nothing is wanting, as a whole man or a whole box. What is true of each particular is true of the whole as such—the whole is that of which nothing is outside. On the other hand that from which something is absent and outside, however small that may be, is not "all." "Whole" and "complete" are either quite identical or closely akin. Nothing is complete (τέλειον) which has no end (τέλος); and the end is a limit. (*Phys*. 207a7-15)

But processes are not incomplete simply because some of their temporal parts are missing. Processes are incomplete because they are actualities that fall short of a goal state. Now what is unusual about processes that go on and on indefinitely is that they have no goal state. In the passage just quoted, Aristotle argues that what is incomplete always has something outside it, and "nothing is complete (τέλειον) which has no end (τέλος); and the end is a limit" (*Phys.* 207a14-5). The absence of a goal or end, then, is a sufficient condition for a process to always have something outside of itself, which is, in turn, a sufficient condition for the infinite to be suspended in a perpetual state of potentiality. Thus, as Aristotle says about the process of dividing a continuous magnitude, "the fact that the indefinitely extendable process of dividing never comes to an end ensures that this activity exists potentially." (*Metaph.* book Θ chapter 6 1048b14-7)

4.8 Existing potentially as matter does

I turn now to an aspect of Aristotle's account of the potential infinite that is ignored by the traditional interpretation. At *Physics* 206b14-6, Aristotle says that "[The infinite] exists potentially as matter exists, not independently as what is finite does." If the point is just that the infinite does not have the ontological independence of a substance, we have seen this theme before, when Aristotle claimed that the infinite exists as a process exists, and one might think that Aristotle is merely making the same point with a different analogy. But he elaborates by telling us that the infinite "is the matter of the completeness which

belongs to magnitude." Since Aristotle takes a magnitude to be a bounded extension, or a form/matter composite consisting of a bounding surface (form) and a spatial extension (matter),[17] the matter of the completeness which belongs to magnitude would seem to be just the matter that is contained and limited by a form. And indeed, Aristotle tells us that the infinite is contained, as matter is contained by form, and it is a part in the way that matter is a part of a form/matter composite. According to the analogy, then, the infinite is unbounded, or ἄπειρος, in the sense that it is unlimited by anything *intrinsic* to it. But it is also "a whole and limited; not, however, in virtue of its own nature, but in virtue of what is other than it," viz., the form of a form/matter composite. In this sense, it is potentially, but not actually a whole, and potentially, but not actually infinite.[18]

What this means, in more concrete terms, I believe, is that potential infinity is a property of the material element in a form/matter composite, such as a bounded magnitude. Or rather, it is a property that the material element contributes to the form/matter composite, viz., its infinite divisibility, or conversely, its being filled out by a potentially infinite number of material parts.

4.9 Existing potentially as both processes and matter do

At first sight, we seem to have competing accounts of potential infinity, one where infinity exists as a process, and one where it exists as a property of matter. Recent commentators have generally tried to promote one of these accounts at the expense of the other. Hintikka, who favors the traditional interpretation, claims that the analogy of infinity to matter is a remnant of a superseded earlier line of

thinking.[19] Jonathan Lear takes the opposite extreme by locating potential infinity entirely in the "structure of the magnitude," and demoting process to the role of merely "bearing witness" to the potential infinite.[20]

But potential infinity is not predicated solely of a process or solely of a magnitude. That is, it does not pertain only to the structure of a magnitude, or only to the nature of a process that divides it. There is a potentially infinite number of material parts and a potentially infinite number of acts of division. Or, perhaps, a more accurate way to say it is that there can be an ever larger number of divided parts as well as an ever larger number of acts of division, and it is clear from the following passage that the possibility of the former depends upon the possibility of the latter:

> But in the direction of largeness it is always possible to think of a larger number: for (γάρ) the number of times a magnitude can be bisected is infinite. Hence this infinite is potential, never actual: the number of parts that can be taken always surpasses any assigned number. But this number is not separable from the process of bisection, and its infinity is not a permanent actuality but consists in a process of coming to be, like time and the number of time. (*Phys*. 207b10-15)

The infinity of the number of parts is inseparable from the infinity of the number of acts of divisions because infinite divisibility is a property of a magnitude that is *defined* in terms of a process of division. The potential infinity of the number of parts relies on a process of coming to be in the sense that it depends on the possibility of the process of division going on and on. And, at the same time, the possibility of the process of division going on and on can be said to rely on the structure of the magnitude to supply it with a potentially infinite number of points of division.

4.10 Goalless changes?

I have suggested that potential infinity exists as a process exists, and in particular, a process that has no goal or end. But how can this be? In *Physics* book 3 chapter 1, Aristotle seems to define change as such as something that involves a goal or τέλος. Aristotle even goes to some trouble in *Physics* book 6 chapter 10 to prove that there can be no infinite or goalless changes (cf. *Phys*. 8.2 252b7-12). The potential associated with an occurrent process is the potential to reach some specific goal, not the potential to go on indefinitely.

But perhaps the infinite "processes" that do go on and on indefinitely can be classed as accidental changes and, as such, will fall outside the class of teleological change considered in the rest of the *Physics*. Perhaps the procession of time owes its lack of teleology to the fact that time is at once a property of all teleological change, and thus lacks any teleology of its own. It is an accidental and goalless composite property of all of the changes in the universe. Likewise, the propagation of mankind as a species is perhaps the accidental sum of the teleological strivings of all individual men and women, and as such, it has no ultimate goal of its own either.

4.11 καθ' αὑτὸ συμβεβηκότα

Dividing a magnitude according to a geometric sequence such as 1/2, 1/4, 1/8, …, $1/2^n$, … (n = 1, 2, 3, …) is different, however, since it is clearly a genuine change. But one will find, as a matter of fact, that any attempt to accomplish this task will

end after a finite number of divisions. Perhaps, then, the potential infinity of the task is reflected in the *counterfactual* possibility that the task could have gone on longer. And perhaps one might generalize to say that all genuine changes are, in fact, finite, but it is just an accidental property of some of them that they could, counterfactually, have gone on indefinitely. This view finds support in *Physics* book 5 chapters 4 and 5, where Aristotle refers to the potential infinite first as a συμβεβηκὸς καθ' αὑτό of number and magnitude (*Phys*. 203b33), and then, equivalently, as a καθ' αὑτὸ πάθος τι (*Phys*. 204a18-19).[21] The concept of a συμβεβηκὸς καθ' αὑτό is introduced at *Metaphysics* 1025a30-1, as "all that attaches to each thing in virtue of itself but is not in its essence, as having its angles equal to two right angles attaches to the triangle." A triangle is presumably defined as "a three angled figure," and this is its essence. But certain other properties not in the definition of a triangle may be deduced from this, which hold eternally and necessarily, e.g., that its angles sum to two right angles, and these properties are συμβεβηκότα καθ' αὑτό. Likewise, since Aristotle defines change in terms of definite goal states, the processes of dividing a magnitude and counting its divisions are, like all processes, essentially finite. But perhaps it is a συμβεβηκὸς καθ' αὑτό of some of these processes that they could, counterfactually, have gone on indefinitely. And perhaps it is the structure of the magnitude being divided that gives these processes this peculiar property.

What Aristotle actually says is that potential infinity is a καθ' αὑτὸ πάθος τι of number and magnitude in a manner analogous to the way in which speech is incidentally invisible (*Phys*. 204a14-17). Just as "the invisible is not an

element in speech, though the voice is invisible," so the infinite is not an element in number or magnitude, though number and magnitude are infinite. The invisible is not an element in speech in the sense that invisibility is not part of the definition of speech, yet speech is, of course, invisible, and necessarily invisible. Likewise, if it is in the essence of a magnitude to be a bounded extension, or a form/matter composite consisting of a bounding surface (form) and a spatial extension (matter), perhaps it is a καθ' αὑτὸ πάθος τι of a magnitude to have a potentially infinite number of material parts, i.e., a property which is not specified in the definition of magnitude, but is deducible from the mention of matter in its definition. Similarly, if it is in the essence of each and every number to be some finite and countable plurality of units, perhaps it is a καθ' αὑτὸ πάθος τι of each such number to be a member of the class of numbers (referred to generically as "number," instead of "a number") that can be increased indefinitely.

4.12 Why potential infinity must be a συμβεβηκὸς καθ' αὑτό of number and magnitude

Thus, taking potential infinity as a συμβεβηκὸς καθ' αὑτό seems to explain the infinite increasibility of number and the infinite extendibility of certain types of changes. But given that infinity is a property of number and magnitude, and given the way that Aristotle defines these things, it is inevitable that infinity must be this sort of property. Aristotle clearly thinks that infinity exists as a property. *Physics* book 3 chapter 5 proceeds as a disjunctive syllogism: The infinite either exists as a substance or as a property. The infinite does not exist as a substance. Therefore, the infinite exists as a property (κατὰ συμβεβηκὸς ἄρα ὑπάρχει τὸ

ἄπειρον (*Phys*. 204a29-30)), and, indeed, as a property of number and magnitude (*Phys*. 204a18-9). Since number and magnitude are the sole members of the genus quantity (*Metaph*. 1020a7-14), infinity is a property of quantity. Quantity is, by nature, measurable or countable because it is divisible into units by which it is measured or counted, viz., parts which are "by nature a 'unit' and a 'this'" (*Metaph*. 1020a7-32; 1057a2-4). This is why Aristotle says that a quantity is always "a particular quantity, e.g., two or three cubits; quantity just means these" (*Phys*. 206a3-5). Since it is impossible to count an infinity of units, and since quantities are, by definition, measurable or countable, there is no infinite quantity (*Phys*. 204b8-10; 204a28-9). But how can Aristotle claim that infinity is a property of number and magnitude if number and magnitude cannot be infinite? To say that there is no infinite quantity is also to say that there is no quantity in whose definition the term "infinity" appears, or as Aristotle puts it, "if the infinite is not a substance, but an accident, then it cannot be, *qua* infinite, and element in things" (*Phys*. 204a14-16). Infinity, then, must be a property of number and magnitude which does not appear in the definitions of number and magnitude, and it must be a necessary and eternal property, since it is a necessary and eternal fact that number does not give out in thought, and that continuous magnitudes are infinitely divisible. Infinity must be, in other words, a συμβεβηκὸς καθ' αὑτό of number and magnitude (*Phys*. 204a29-30; cf. *Phys*. 204a14 and 28-29).[22]

5.0 THE INFINITE DIVISIBILITY OF MOTION: CONCEPTUAL CONSTRAINTS

The fact that Aristotle takes quantity to be essentially measurable or countable puts a constraint on the way he conceives of motions as things having quantities. For instance, Aristotle believes that if it were necessary, in order to give an account of the quantity of a motion, to also give an account of an infinite quantity of submotions into which the motion is divisible, then the quantity of the motion would be unknowable. This is because quantities are known by either measurement or by counting (*Metaph*. 1052b20; 1053a7-8), and to give an account of an infinite quantity of submotions would involve measuring and counting each of the motions in turn. Since this is impossible, then the quantity of the motion must be unknowable.

This constraint on the way Aristotle conceives of motions as things having quantities can be seen at work in his discussion of Zeno's dichotomy paradox. Aristotle concedes to Zeno at *Physics* 233a26-8 that motion would indeed be impossible if, in order to run the race, the runner would need to "come in contact with things infinite in quantity (κατὰ τὸ ποσόν)," but, as Zeno fails to realize (and this is Aristotle's point), the reason for this is that there can be no things "infinite in quantity" to come in contact with.

The term "quantity" (τὸ ποσόν) is significant, since for Aristotle, quantities are, by nature, measurable or countable. Thus, a thing must be measurable or countable if it is to be called a quantity, and if it is neither measurable nor countable it is not a quantity. At *Physics* 263a4-b8, Aristotle attempts to establish that the reason why it is impossible to come "in contact with

things infinite in quantity" is that there can be no "things infinite in quantity" to come in contact with, and he does this by reducing the term "infinite in quantity" to absurdity. Aristotle supposes, *per impossibile*, that the infinite collection of the run's subsegments is a quantity, i.e., something that is definite, countable, and knowable, then he imagines an attempt to verify this by means of a process of dividing the motion into an infinite number of units and counting them. Failing at this, he concludes that the infinite collection of the run's subsegments is *not* a quantity, and, therefore, that it is impossible to come "in contact with things infinite in quantity."

Thus, since an infinite quantity cannot be measured or counted, and one must measure or count a quantity if one is to give an account of it, if a motion had an infinite quantity, one could neither give an account of it nor know it. And insofar as having a quantity is an essential property of a given motion, if the quantity of the motion were unknowable, then the motion itself would be unknowable, but given Aristotle's assumption that quantities are knowable, there are no such motions.

5.1 What is real must be thinkable

One might object that Aristotle has simply legislated this result by defining a quantity as something that is countable or measurable. But if this reflects a prejudice, Aristotle is at least consistent about it. As Jonathan Lear points out, "Throughout Aristotle's work this theme recurs: the possibility of philosophy—of man's ability to comprehend the world—depends on the fact that the world is a

finite place containing objects that are themselves finite."[23] This attitude can be seen clearly at *Posterior Analytics* 82b37-9, where Aristotle argues that accounts of the essence of a substance must be finite because the essential predicates of a substance are finite, and this follows from the facts that "definition is possible, or in other words, […] essential form is knowable, and an infinite series cannot be traversed."

Aristotle's commitment to a knowable and finite world, is also reflected, as Hintikka notes, in the fact that "realisability in thought" is a necessary condition for "actual physical realisability."[24] Thus, if thought is knowable and finite, so is the world. Hintikka points out that in *De anima* 3.6, 431a1-2, Aristotle claims that "the thinking mind is formally identical with the object of which it is thinking," or in Aristotle's words, "Actual knowledge is identical with its object." So for Aristotle, the conceivability of something also entails its actualizability *qua* form. This does not mean that conceiving of a unicorn or of a "goat-stag" entails that it can be instantiated in a form-matter composite. It merely entails that it can be instantiated in the mind.

Two additional principles go a long way toward ensuring that physical realizability presupposes mental realizability, both formally and materially. First, in the case of artifacts, the principle of synonymy advanced in *Metaphysics* book Z chapters 7-9 ensures that a form in the mind is no less actual than a form in an artifact, since the source and cause of the form in an artifact is a form in the mind. Second, in the case of geometrical objects, not only is the geometer's thinking an "actuality" (*Metaph.* book Θ chapter 9 1051a30-1) in that his mind is formally

identical with the form of geometrical objects, but (and Hintikka does not note this) intelligible matter provides an analogue in thought to sensible matter, so that geometrical objects can have not only the formal characteristics, but also the material characteristics of physical objects (*Metaph*. 1036a9-12, 1037a2-5).

It is controversial whether intelligible matter exists in the world as well as in the mind, or strictly in the mind. For my purposes here, it only matters that, in any event, it exists in the mind. Mueller, Annas, and Hussey have argued that, due to the discrepancy arising, for instance, "when one draws a line on the ground and calls it a foot long when it is not" (*Metaph*. 13.3 1078a19-20), mathematical objects must be created by the thought of the mathematician out of intelligible matter, and this makes them mind-dependent and cut off from the material world. Jonathan Lear, however, argues, in "Aristotle on Mathematics," (pages 175-183), that at *Physics* 193b23-194a12 and *Metaphysics* book 13.3, 1077b18-1078a31, Aristotle holds that the geometers study physical objects but not *qua* physical objects, so that, the mathematical objects are the physical objects under some description. It seems that the correct way to reconcile the views of Lear on one hand and Mueller, Annas, and Hussey on the other, is to take Aristotle to countenance *both* mind-dependent and mind-independent geometrical objects. Thus Aristotle is not talking about the same activity "when one draws a line on the ground and calls it a foot long when it is not" and when one studies a body *qua* a foot long. In other words, when there are perfect circles instantiated in the universe, such as a heavenly body, for instance, the geometer studies the body *qua* geometrical object. However, when such a perfect instantiation is not at hand, it

is possible for the geometer to use any available imperfect circle as a symbolic proxy for a circle that he creates mentally in intelligible matter. In fact, since at *Metaphysics* 1036a11, Aristotle says that intelligible matter is "present in sensible objects but not insofar as they are sensible," it appears that intelligible matter is not "in" the mind in any literal sense, but seems to represent an ability of the mind to make a geometrical object out of an imperfect sensible object by correcting the imperfections in it.

5.2 Constraints on the concept of infinity

The key material characteristic for the concept of infinity is, of course, infinite divisibility, and Aristotle tells us at *Metaphysics* book Θ chapter 9 1051a21-31 that the infinite divisibility of intelligible matter is essentially equivalent to the infinite divisibility of sensible matter. This is one of the reasons why Aristotle tells us in his analysis of Zeno's paradox that "counting out halves is no different from dividing into halves" (*Phys*. 263a25-6), viz., a mental act of division is metaphysically commensurable to a physical act of division. Moreover, a mental act of division is numerically equivalent to a mental act of counting when the purpose of the division is to divide a magnitude into units for the purpose of measuring it.

A negative aspect of assuming that physical realizability presupposes mental realizability is that our intuitions about physical objects will tend to constrain the way we conceive of things like infinity. If our concept of infinity is based on the infinite divisibility of a magnitude, and the infinite divisibility of a

magnitude is conceived of in terms of a process of counting divisions, or more generally, in terms of a process of going step-wise through a sequence that has some first element, then we will naturally be limited to thinking of infinity as something untraversable, as Bertrand Russell points out:

> The difficulty, like most of the vaguer difficulties besetting the mathematical infinite, is derived, I think, from the more or less unconscious operation of the idea of *counting*. If you set to work to count the terms of an infinite collection, you will never have completed your task. … But it is not essential to the existence of a collection, or even to knowledge and reasoning concerning it, that we should be able to pass its terms in review one by one. This may be seen in the case of finite collections; we can speak of "mankind" or "the human race," though many of the individuals in this collection are not personally known to us. We can do this because we know of various characteristics which every individual has if he belongs to the collection, and not if he does not. And exactly the same happens in the case of infinite collections: they may be known by their characteristics although their terms cannot be counted. In this sense, an unending series may nevertheless form a whole, and there may be new terms beyond the whole of it.[25]

The "whole" in the last sentence presumably refers to an infinite collection with a Cantorian transfinite cardinality of $\aleph_0$ or higher. The least infinite limit cardinal $\aleph_0$ and the corresponding least infinite limit ordinal ω_0 represent the possibility of passing through the whole of a denumerably infinite sequence and reaching "new terms" (such as ω_1, $\omega_1 + 1$, $\omega_1 + 2$, etc.) beyond it, despite the fact that the sequence has no last term. This possibility, which Aristotle expressly denies when he says that "Nothing is complete (τέλειον) which has no end (τέλος)" (*Phys*. 207a14), is only made intelligible by the abandonment of counting the members of a denumerably infinite collection in favor of making generalizations over them (e.g., that the members of a collection with a cardinality

of $\aleph_0$ may be paired off with the natural numbers while the members of a set with a cardinality of $\aleph_1$ or higher may not).

One might argue that it is inappropriate to make counting an infinite sequence equivalent to traversing it, since it is possible to imagine a counterexample of an infinite sequence which cannot be counted, but which can, nonetheless, be traversed. If one assumes, as Aristotle does, that the world had no beginning, one must admit the existence of a *beginningless* traversal of an infinite sequence of celestial rotations, as Philoponus claims in his *De aeternitate mundi contra Proclum*.[26] But one cannot even begin to count this sequence, since counting requires a first item to be counted, and there is no first celestial rotation by assumption. But Aristotle, perhaps mistakenly, as I argued in Sections 4.2 and 4.3, would think that he could rule out this counter-example by pointing out the fact that past years and celestial rotations no longer exist, and, therefore, do not constitute an actual infinity. Therefore, I see no reason not to equate the Aristotelian notion of traversal with counting, or at least, more generically, with a process of going step-wise through a sequence that has some first element, even though, perhaps, he should not have thought of traversal in this way, given the problem raised by Philoponus.

5.3 Constraints on the concepts of motion and spatial magnitude

Limitations on the concept of infinity, in turn, limit the concepts of motion and spatial magnitude, in regard both to their analyzability and to the manner in which we must apprehend them. Aristotle holds that the concept of a denumerably

infinite sequence as a traversable whole is inconceivable — there is no form of infinity that the mind can grasp. So, assuming that what is real must be conceivable, if the parts of a physical magnitude are infinite, they cannot be formal parts. Hence magnitude must be apprehended as formally simple (or, at least, formally finite) so that its form is not analyzable into things like infinite point-sets. And since kinetic magnitude is formally derived from spatial magnitude,[27] motion is not formally analyzable into motion segments or into being at certain positions at certain instants in time (i.e., the "at-at" ontology of motion).[28]

5.3.1 The apprehension of a magnitude

Since the infinite parts of a magnitude are material and not formal, and a magnitude is bounded by a single unifying form, a magnitude will be apprehended all at once, rather than piecemeal, i.e., by means of apprehending its parts. This doctrine is present in the passage immediately preceding the line I just quoted:

> For [the account of a line is not infinite], to whose divisibility there is no stop, but which we cannot think if we do not make a stop (for which reason one who is tracing the infinitely divisible line cannot be counting the possibilities of section), but the whole line also must be apprehended by something in us that does not move from part to part.—Again, nothing infinite can exist; and if it could, at least the notion of infinity is not infinite. (*Metaph*. 2.2 994b23-7)

That "the whole line" is "apprehended by something in us that does not move from part to part" means that it is apprehended all at once instead of piecemeal. The same idea is further elaborated at *De anima* 430b6-20:

> Since the undivided is twofold, either potentially or actually, nothing prevents one thinking of the undivided when one thinks of a length (for this is actually undivided), and that in an undivided time; for the time is divided and undivided in a similar way to the length. It is not possible to say what one was thinking of in each half time; for these do not exist, except potentially, if the whole is not divided. But if one thinks of each of the halves separately, then one divides the time also simultaneously; and then it is as if they were lengths themselves. But if one thinks of the whole as made up of both halves, then one does so in the time made up of both halves. That which is thought and the time in which it is thought are divided incidentally and not as those things were; for there is in these too something undivided, although surely not separate, which makes the time and the length unities. And this exists similarly in everything which is continuous, both time and length. (*DA* 430b6-20)

The point, here, seems to be that since a line is a form/matter composite, it is divisible potentially, but the mental act that grasps the form of the line, i.e., "something undivided, although surely not separate, which makes the time and the length unities," is indivisible. One does not apprehend the form of a line by mentally traversing it, so that part of it is apprehended first, and another part second. As at *Metaphysics* 1051b24 ff., the mental act of apprehending a form is non-discursive and instantaneous, like grasping or touching — it happens all at once. One may contemplate a form for a period of time, of course, but the act of beginning to contemplate it is instantaneous. One might think of a half length first, and then the other half second, but this amounts to two mental acts instead of two half mental acts. One might also think of a line as divisible, but, since divisibility is an incidental property of a line, Aristotle would argue that the form of the line has already been grasped prior to the predication of "divisibility," which is just the serial concatenation of the concepts "line" and "divisibility."[29]

5.4 Dividing up a motion

The fact that motions are to be apprehended all at once has a specific implication for how we must conceive of the infinite divisibility of motions. Aristotle argues in *Physics* 5.4 that motions for a given moving body are numerically distinct if and only if they are divided by periods of rest, so whereas in the case of spatial magnitudes a division is an act of cutting, in the case of a motion a division is an act of coming to a halt. But since magnitudes are to be apprehended all at once, instead of piecemeal, what we are envisaging when we talk about the infinite divisibility of a motion is the *counterfactual* possibility of having stopped at any of an infinite number of places along the course of a journey. This is because, in order to apprehend a motion all at once, we must imagine it in the perfect tense. The process of infinitely dividing a motion, then, will be a process of imagining a series of pauses that might have been made on a given journey, but have not. Since thought is as infinitely extendable as change and time (*Phys*. 208a20-1), a motion may be divided *ad infinitum* as well as a spatial magnitude can.

6.0 THE INFINITE DIVISIBILITY OF MOTION: ARISTOTLE'S SOLUTION

We have seen that Aristotle believes that if it were necessary to admit that in performing a motion one has to perform and infinity of sub-motions, then the motion would be impossible. But Aristotle *does* allow that one may pass through "a [potentially] infinite number of half-distances" incidentally but not essentially in the course of a continuous motion. "For though it is an incidental characteristic of the distance to be a [potentially] infinite number of half-distances, this is not its

real and essential character" (*Phys*. 263b7-9).[30] What I take this to mean is that it is unnecessary, in order to move from point A to point B, also to complete an infinite quantity of submotions into which the motion is divisible. This is because it is not a necessary condition for completing a continuous motion to complete an infinite sequence of sub-motions. Rather, one may complete a potentially infinite sequence of sub-motions only incidentally, in the sense that after a motion is completed, one may imagine that the motion *had been* segmented as many times as one likes (by means of an indefinitely extendable imaginative process of enumerating, counter-factually, the places at which one might have stopped but did not). But this possibility does not figure into the account of how the motion was actually completed.

This follows straightforwardly from the fact that the parts of the distance traversed in the sub-motions are material parts of the total distance traversed, and that only formal parts are salient for definition and understanding (*Metaph*. 1035b31 ff.). This seems perfectly reasonable in the context of Zeno's dichotomy paradox, where not only the choice of segmentation seems arbitrary, but the segmenting itself seems optional and *ex post facto*.[31] This is because, in the dichotomy paradox, segmentation of the time, the distance, and the motion does not enter into, and is not implicit in, the motion as it is described — as a simple traversal of a distance without a reference to any other properties of the motion. In fact, Aristotle's solution to the dichotomy paradox *relies* on the fact that the temporal and spatial magnitudes need not be segmented *at all* to make sense of the motion of the runner. Aristotle and Zeno, of course, are free to make different

assumptions about what it means for the segmentation to be optional. Zeno can hold that segmentation is optional because no matter whether one chooses to segment or not, the magnitude is, in a sense, already segmented. Aristotle, on the other hand, can take precisely the opposite view, that segmentation is optional because the magnitude is not, and need not be segmented. According to Aristotle, if one chooses not to divide up the magnitude, one need not recognize the traversal of its parts. And if one chooses not to (or cannot) divide up the magnitude into an infinite number of parts, one need not (or must not) recognize the traversal of an infinite number of parts.

7.0 A DIFFICULTY

In Section 1.6, I suggested that we might achieve Plato's objective of explaining what is involved in arriving at and departing from the instant of change, and address Aristotle's mismatch between spatial and kinetic termini, by imagining that there is no first or last *non-instantaneous* velocity above zero. But since this requires the moving body to pass through an infinite decreasing sequence of *non-instantaneous* velocities, this option would appear to be ruled out, if the sequence of velocities is an actual infinity, and since, as we have seen, Aristotle does not countenance actual infinities.

Two considerations would seem to indicate that this sequence of velocities is an actual infinity. First, velocity would seem to be a real or actual property of a motion. At any rate, it is hard to imagine an argument to the effect that it is not, and as a result, it follows that accelerations and decelerations represent real

changes in the *character* of a motion. Second, as I said before, Aristotle's solution to the dichotomy paradox *relies* on the fact that the temporal and spatial magnitudes need not be segmented *at all* to make sense of the motion of the runner. But this is not the case if we want to invoke an infinite sequence of discrete decelerations to explain how the motion passes into and out of existence. These decelerations must be accounted for if they are to figure into the account of how the motion comes to be or passes away. If one is committed to the claim that there is no first or last non-instantaneous velocity above zero, then, unless one acknowledges the traversal of an infinite sequence, one will have the paradoxical result that a moving object will never come to a stop — its velocity will just keep decreasing *ad infinitum*.

On the other hand, if in coming to a stand, a moving body can only go through a finite number of decelerations, then it seems that Aristotle's only recourse is to conceive of the ultimate stage of the transition between motion and rest, i.e., the coming to be or passing away of motion, as being discontinuous, or as involving a step-change in velocity (i.e., where there is some last positive non-instantaneous velocity before rest).[32] In other words, however much Zeno's runner slows down on his approach to the finish line, Aristotle must hold that his ultimate coming to rest is abrupt and instantaneous. The same phenomenon will happen in reverse, of course, at the beginning of a change, viz., there must also be a step-change in velocity at the inception of a motion. In order to reach a period of zero velocity, either a moving body must traverse the whole of an infinite sequence of non-instantaneous velocities, or it must traverse a finite sequence of

non-instantaneous velocities and then transition to the period of zero velocity discontinuously. But recognizing the traversal of an actually infinite series is not really an option because it would violate one of Aristotle's most basic philosophical commitments — a commitment that finite human minds are up to the task of understanding the universe, because the universe, in its essence, is a finite place containing finite things. So Aristotle would appear to have no choice but to recognize a discontinuous change in velocity at the beginning and end of a motion.

Now if something is in motion if and only if it has a positive velocity, and the acquisition or loss of a positive velocity constitutes the coming to be or passing away of a motion, a discontinuous change in velocity at the beginning or end of a motion will constitute a discontinuity in its coming to be or passing away. And as it happens, Aristotle claims that there is no process of coming to be in motion in *Physics* book 5, chapter 2, on the grounds that it involves an infinite regress:

> Again, if there is to be change of change and becoming of becoming, we shall have an infinite regress. Thus if one of a series of changes is to be a change of change, the preceding change must also be so: e.g. if simple becoming was ever in process of becoming, then that which was becoming simple becoming was also in process of becoming, so that we should not yet have arrived at what was in process of simple becoming but only at what was already in process of becoming in process of becoming. And this again was sometime in process of becoming, so that even then we should not have arrived at what was in process of simple becoming. And since in an infinite series there is no first term, here there will be no first stage and therefore no following stage either. On this hypothesis, then, nothing can become or be moved or change. (*Phys*. 225b33-a6)[33]

Of course, acceleration is a "change of a change," and a less than generous reading of this passage might take it to deny the existence of acceleration altogether, but I am inclined to think that it is only meant to deny acceleration from a stand, insofar as this is equivalent to the coming to be of a motion.[34] Thus, we may speculate that Aristotle was aware that a step-change in velocity at the beginning and the end of a motion was required on the assumption that the traversal of an actual infinity is impossible.

There are two reasons why *we* might be disconcerted with step-changes in velocity, but they would not have bothered Aristotle. First, from a modern perspective, i.e., one that assigns a sense to instantaneous velocity, step-changes in velocity look paradoxical in that it appears as though the decelerating object has two velocities at once. This is not a problem for Aristotle since an object cannot have an instantaneous velocity. The second reason has to do with Newton's Second Law of Motion, which effectively prohibits step-changes in velocity by making force directly proportional to acceleration. If acceleration were infinite, then an infinite force would be required to cause it, but an infinite force is impossible. Aristotle does not have this problem, however, since in his discussion of dynamics in *Physics* 7.5 he makes what we would call force (δύναμις) directly proportional to velocity instead of acceleration (which makes his notion of δύναμις in a local motion analogous to Newton's definition of "quantity of motion"). For Aristotle, δύναμις is needed to sustain as well as to initiate motion, and δύναμις must be continuously applied as the object moves; more δύναμις if the object is moving faster, less δύναμις if it is moving slower.

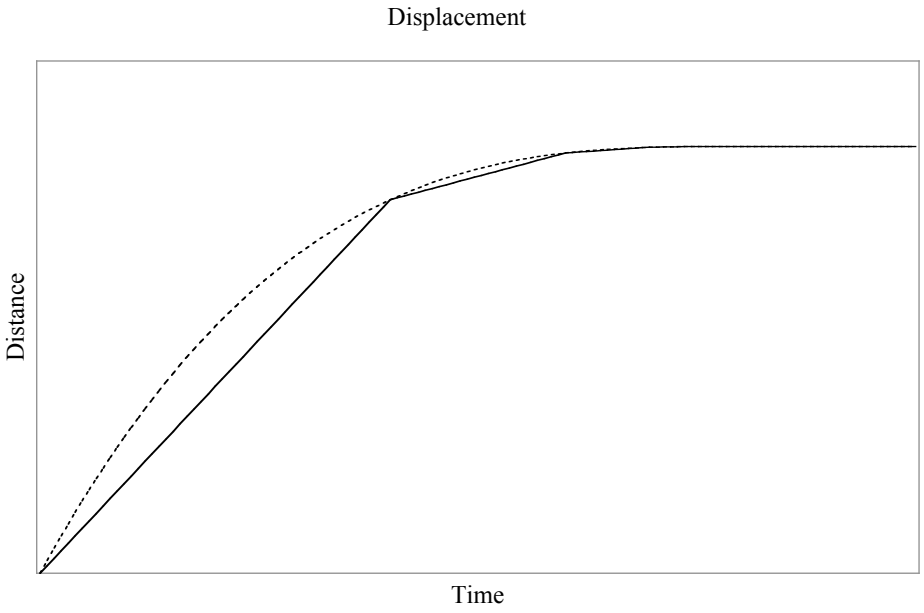
But with no concept of inertia, however, change in velocity will be as sudden as the application and the withdrawal of a δύναμις.[35] And, if the application of δύναμις is as sudden as the coming in contact of the mover with the thing moved (see *Cael*. 280b6-9), then, by this reasoning also, the change in velocity at the beginning and end of a motion will be instantaneous.

8.0 A MORE FORMAL ACCOUNT OF HOW ARISTOTLE MIGHT HAVE CONCEIVED OF THE PHENOMENON OF COMING TO A STAND

In the following section, I will sketch a more formal description of how Aristotle might have conceived of the phenomenon of coming to a stand using certain features of Archimedes' *Quadratura Parabolae* as a model. I use the phrase "might have" counterfactually, both in the sense that Aristotle apparently did not attempt such an exposition, and that, even if he did, he probably would not have couched his account in the more formal idiom of a geometer. Nonetheless, I believe that it may be *permissible*, because, as I argued in Section 3 above, Aristotle's mathematical finitism, and his conception of infinite series are consistent with, if not informed by, 5th and 4th century attempts at quadrature, of which Archimedes' attempt, though it post-dates Aristotle, is still representative. I believe that it may be *desirable* to attempt such an exposition since it will clarify exactly what is implied by the combination of Aristotle's concept of motion and his strong mathematical finitism.

Archimedes proves that for any finite $n \geq 1$, the area A bounded by a straight line and a section of a right-angled cone is equal to the series $A + A/4 + A/16 + A/64 + \ldots + A/4^n + 1/3(A/4^n) = 4A/3$. Notice that the *n*th member of the

series contains the plug $1/3(A/4^n)$, which bridges the gap between the area of the nth partial sum and the area under the right-angled cone. Let us, for the sake of illustration, take a case of coming to a full stand over a given unit of distance, and assume that to the infinite sequence of non-overlapping subintervals of time 1/2, 1/4, 1/8, …, $1/2^n$, … (n = 1, 2, 3, …), there corresponds the sequence of non-overlapping subintervals of distances 7/8, 7/64, 7/512, …, $7/8^n$, … (n = 1, 2, 3, …)[36], which results in the sequence of velocities 7/4, 7/16, 7/64, …, $7/4^n$, … (n = 1, 2, 3, …). Normally, this would imply that as n approaches infinity, the non-instantaneous velocity of the moving body within each subinterval of time ($v_n = 7/4^n$) approaches 0 as a limit, while the sequences of partial sums of the times (expressed by $S_n = (2^n\text{-}1)/2^n$) and the distances (expressed by $S_n = (8^n\text{-}1)/8^n$) each approach 1 as a limit. But we can make the series finite by introducing plugs at the end, as Archimedes does. Thus, the finite time series would be: $1/2 + 1/4 + 1/8 + \ldots + 1/2^n + 1/2^n$ for any finite $n \geq 1$. The finite distance series would be $7/8 + 7/64 + 7/512 + \ldots + 7/8^n + 1/8^n$ for any finite $n \geq 1$. And finally, the velocity sequence would be $1/4^n$, i.e., 7/4, 7/16, 7/64, … $7/4^n$, $1/4^n$ for any finite $n \geq 1$. These sequences describe a rectilinear displacement plot, which will remain rectilinear as long as n is finite, as the following graph illustrates:



And since velocity is constant over a linear segment of a displacement plot, the velocity will decrease abruptly at each of the points where the segments join. But the change that interests us is the last step-change, since it must occur at the instant dividing the periods of motion and rest. In my example, the moving object will decelerate instantaneously at this instant from a velocity of $1/4^n$ to a velocity of 0.

9.0 DISCONTINUOUS CHANGE IN ARISTOTLE

Sorabji points out that Aristotle seems to recognize only four kinds of "genuine" or non-incidental changes, viz., change of quality, change of place, change of size, and generation and destruction. Sorabji cites *Physics* 3.1, 200b32-201a16, *Physics* 5.2, and *Metaphysics* 11.12. He also notes that in each of the four kinds

of "genuine" changes, change is alleged to be continuous, citing *Physics* 6.6, 237a17-b3 and b9-21. Yet Aristotle seems to allow a variety of exceptions to this rule. At *De sensu* 446b28-447a6, Aristotle allows that some qualitative changes may, but are not required to occur all at once, as in the simultaneous freezing of all of the parts of a pond. And a few lines earlier, at *De sensu* 446a18-20, he admits the discontinuity of transitions between colors, tastes, and sounds, due to what he assumes is the limited number of discriminable colors, tastes, and sounds in existence. He also allows discontinuity in the force needed to move an object, positing a threshold force, below which a motion will not occur (due to friction, presumably) (*Phys*. 7.5 250a15-19). But, by far, the largest number of discontinuous changes are found under the heading of "generation and destruction," in the form of comings to be or passings away that do not require a process.

Among the things that Aristotle allows to come to be and pass away in this manner, one can make a very rough distinction, i.e., some of them are indivisibles while the rest are relations. Things may stand or fail to stand in relations to other things, but there is no process of coming to stand in a relation, or coming not to stand in a relation (*Phys*. 225b11-13; 246b11-12; 247b4). Relational changes are accidental (*Phys*. 225b11-13), and, indeed, the coincidence of states of affairs, since coincidence is a sort of relation, comes into and go out of existence in a like manner (*Metaph*. 1026b22; 1027a29).

And if we construe indivisibility very broadly, I believe that we can include as indivisibles, in addition to things such as lines, points, surfaces, and

instants, things like forms and ἐνέργειαι. Physically indivisible things like lines, points and surfaces pass in and out of existence without a process of transition because they have no parts to allow them to be in both the *terminus a quo* and *terminus ad quem* while the object is changing, and Aristotle insists on this as a condition of continuous change (*Phys*. 234b10-20). But it is the same with forms and ἐνέργειαι such as pleasure, seeing, and knowing. Substantial forms cannot come into or go out of existence piecemeal (*Metaph*. 1039b26; 1043b14; 1044b21), since the unity of a substance is irreducible to the unity of any of its parts. ἐνέργειαι, on the other hand, cannot come into being piecemeal because they involve no distinction between a *terminus a quo* and *terminus ad quem*. They are whole and indivisible, like points and units since they have no parts (i.e., phases) which could be missing (*EN* 1174b10-13). Processes like taking a walk, however, assuming that the walk has some destination, *do* have parts (phases) which must come into existence serially for the walk to be complete, and this is why Aristotle insists that one cannot complete a walk without walking (*Phys*. 232a10-11).

10.0 CONCLUSION

But what about the coming to be and passing away of motion, which has been the main focus of this chapter? Does it fit into either of these categories? Sorabji and Niko Strobach make a distinction which I believe can be used to suggest that the coming to be and passing away of motion might be a relational or accidental change. Sorabji makes a distinction between changes involving what Strobach

has later dubbed "comparative" and "non-comparative" properties.[37] Non-comparative properties are termini of the four kinds of genuine change — qualities, positions, sizes, and states of existence and non-existence. Comparative properties are "properties which are assigned to an object by comparing its states at different instants between which there is always a certain period of time."[38] Motion and rest are comparative properties. Motion is the property of being in the process of traversing a continuum from one non-comparative property to another. Rest is the property of possessing the same non-comparative property for a continuous period of time. Since motion and rest mention non-comparative changes in their definitions, i.e., a motion is always a motion with respect to quality, place, size, or existence, the change between motion and rest is, in a sense, derivative of non-comparative changes. Perhaps this second order or derivative character would make the change between motion and rest relational and incidental in Aristotle's view.

Or perhaps, while change between motion and rest is discontinuous in one sense, it is continuous in another. Perhaps the derivative character of motion and rest would give changes between the two what Aristotle calls a "not *per se* continuity" (τὸ μὴ καθ' αὑτὸ συνεχές, *De sensu* 445b21-9; 446a16-20), which Sorabji interprets to mean, in the case of a discontinuous change between discriminable pitches, for instance, "that a change to the next discriminable pitch, in the *discontinuous* series of discriminable pitches, may proceed by a *continuous* movement of a stopper along a vibrating string."[39] Aristotle certainly does think that the continuity of change of place is derived from the continuity of place.

However, there is an important disanalogy with the case of the moving stopper, which I mentioned in Section 1.4 — whereas the discontinuous change in discriminable pitch coincides with the arrival of the stopper at a particular position along the vibrating string, the transition from motion to rest does *not* coincide with the arrival of the train at the station. At the instant of arrival, the train is not yet in a state of rest because rest takes time.

Now it may be tempting to conclude, based on the foregoing, that Aristotle has a problem, but it is a rather minor one that may be classed as just an isolated exception to his theory. But this appearance is false, since admitting discontinuity into the heart of his metaphysics of motion is just as objectionable as conceding that an actually infinite series is traversable. This is because, if passing to and from a period of zero velocity is discontinuous because Aristotle cannot account for how the transition between motion to rest ultimately takes place, so too must the passage to and from any period with some positive velocity. Suppose that a moving body traveling at 6 miles per hour decelerates to a velocity of 5 miles per hour. If the number of decelerations that the body undergoes is finite, how will the body ultimately decelerate to 5 miles per hour without undergoing a discontinuous drop in speed? No matter how close the velocity gets to 5 miles per hour (e.g., 5.1, 5.01, 5.001, 5.0001 m.p.h., etc.), if the number of decelerations is finite, then the ultimate deceleration to 5 miles per hour must be discontinuous. In other words, the problem with stopping and starting is perfectly generalizable to transitions between any two velocities. If this is so, then motions will be discontinuous not only at their beginnings and endings, but at any time at which

they change their speed. It will not do to simply say that a decrease or increase in velocity is continuous if and only if for any period at a given velocity, adjacent periods will differ in velocity by less and less as shorter periods are taken, because, no matter how short a period one takes, the velocity will always differ from the given velocity by some positive amount.

Calling motion a "comparative property," I think, highlights a basic shortcoming of Aristotle's concept of motion. The idea that motion is simply the traversal from place A at time t to place B at time $t + 1$ makes it not only impossible for Aristotle to account for how motions come to be and pass away, but also for Aristotle to conceive of continuous changes in velocity. For Aristotle, the velocity associated with the traversal of an object between spatial termini is always a *non-instantaneous* velocity, and an non-instantaneous velocity always masks the variation in the velocity over the period of change. Aristotle can divide the motion up as finely as he likes, but unless he embraces the concept of instantaneous velocity, there will always be jumps or drops in speed as the object moves from one motion subsegment to the next.

NOTES TO CHAPTER 3

[1] Sorabji, *Time, Creation And The Continuum*, 403-421.

[2] I do not wish to say, here, that Sorabji's solution explicitly addresses this difficulty, but merely that it has the resources to do so.

[3] See *A. Pr.* 2.25 69a31, *Cat.* 7 7b31, and *EE* 2.10 1226a29 for general references and *A. Pst.* 1.9 75b41 (Bryson), *Phys.* 1.2 185a16 (Antiphon), *SE* 171b15 (Hippocrates), *SE* 171b16 (Bryson), *SE* 172a3 (Bryson), *SE* 172a7 (Antiphon).

[4] See section 5.0 below.

[5] For an alternative reading, see Gregory Vlastos, "Zeno's Race Course," *Journal of the History of Philosophy* 4 (1966): 95-108.

[6] Aristotle makes the rather bad argument that since certain geometrical facts become obvious once a suitable figure has been constructed, discovery of a geometrical fact must presuppose the construction of a suitable figure. The fallacy is one of affirming the consequent, i.e., "If a geometrical construction φ has been made, geometrical fact ψ becomes known. Geometrical fact ψ becomes known. Therefore, a geometrical construction φ has been made."

[7] Euclid *Elements* 12.2.33-37: τέμνοντες δὴ τὰς ὑπολειπομένας περιφερείας δίχα καὶ ἐπιζευγνύντες εὐθείας καὶ τοῦτο ἀεὶ ποιοῦντες καταλείψομέν

τινα ἀποτμήματα τοῦ κύκλου, ἃ ἔσται ἐλάσσονα τῆς ὑπεροχῆς, ᾗ ὑπερέχει ὁ ΕΖΗΘ κύκλος τοῦ Σ χωρίου.

[8] Euclid *Elements* 12.2.37-41: ἐδείχθη γὰρ ἐν τῷ πρώτῳ θεωρήματι τοῦ δεκάτου βιβλίου, ὅτι δύο μεγεθῶν ἀνίσων ἐκκειμένων, ἐὰν ἀπὸ τοῦ μείζονος ἀφαιρεθῇ μεῖζον ἢ τὸ ἥμισυ καὶ τοῦ καταλειπομένου μεῖζον ἢ τὸ ἥμισυ, καὶ τοῦτο ἀεὶ γίγνηται, λειφθήσεταί τι μέγεθος, ὃ ἔσται ἔλασσον τοῦ ἐκκειμένου ἐλάσσονος μεγέθους.

[9] I follow Michael White's "intuitive and informal" use of the term "converse" here. (Michael J. White, *The Continuous and the Discrete* (New York: Oxford Clarendon Press, 1992), 149). The axiom of Archimedes is as follows: "Of unequal lines, unequal surfaces, and unequal solids, the greater exceeds the less by such a magnitude as, when added to itself, can be made to exceed any assigned magnitude among those which are comparable with [it and with] one another" book 1, "On the Sphere and the Cylinder" in *The Works of Archimedes*, trans. T. L. Heath (Cambridge: Cambridge University Press, 1897).

[10] *In phys*. 9.55.4-8: καὶ κατὰ τὸν αὐτὸν πάλιν λόγον τέμνων τὰς πλευρὰς τοῦ ἑκκαιδεκαγώνου τοῦ ἐγγεγραμμένου καὶ ἐπιζευγνὺς εὐθείας καὶ διπλασιάζων τὸ ἐγγραφόμενον πολύγωνον καὶ τοῦτο ἀεὶ ποιῶν ὥστε ποτὲ δαπανωμένου τοῦ ἐπιπέδου ἐγγραφήσεσθαί τι πολύγωνον τούτῳ τῷ τρόπῳ ἐν τῷ κύκλῳ, οὗ αἱ πλευραὶ διὰ σμικρότητα ἐφαρμόσουσι τῇ τοῦ κύκλου περιφερείᾳ.

[11] Wilbur R. Knorr, "Infinity And Continuity," in *Infinity And Continuity In Ancient And Medieval Thought*, ed. Norman Kretzmann (Ithaca: Cornell University Press, 1982), 112-145.

[12] See section 5.0 below.

[13] Precession is the fact of preceding in time. Infinite precession is the fact of an infinite number of past events preceding one another in time.

[14] Richard Sorabji points out another weakness in Aristotle's strategy: Since Aristotle is willing to countenance collections of objects that exist less than fully (such as points and potential entities), then there is no reason why infinite collections of non-present events or things should escape being actual infinities simply because they are not present. (Sorabji, *Time, Creation And The Continuum*, 216-7.)

[15] William Charlton, "Aristotle's Potential Infinites," in *Aristotle's Physics: A Collection of Essays*, ed. Lindsay Judson (Oxford: Oxford Clarendon Press, 1995), 145.

[16] As Aristotle puts it, "change is the actuality of what exists potentially, *qua* existing potentially" (*Phys*. 201a10-11).

[17] See *Phys*. 209b5. Aristotle identifies unbounded extension with matter at *Phys*. 209b9-10.

[18] The foregoing paragraph paraphrases *Phys*. 207a20-28.

[19] Jaakko Hintikka, "Aristotelian Infinity," *Philosophical Review* 75 (April 1966): 207.

[20] Jonathan Lear, "Aristotelian Infinity," *Proceedings of the Aristotelian Society* 80 (1979): 191.

[21] Cf. *Phys*. 204b30.

[22] In the case of number, this also follows from the fact that numbers are either odd or even and that infinity is neither odd nor even (*Metaph*. 1084a2-4). In the case of magnitude, this also follows from the alleged fact that there is no infinitely large body or collection of bodies of which an infinite magnitude can be a property (*Phys*. 204a34-206a7).

[23] Lear, "Aristotelian Infinity," 202.

[24]Hintikka, "Aristotelian Infinity," 208.

[25] Bertrand Russell, *Our Knowledge of the External World* (New York: W. W. Norton & Company, Inc., 1929), Lecture 6, 182-198.

[26] See section 4.2 above.

[27] See *Physics* 4.11.

[28] Michael White suggests (White, *Continuous and Discrete*, 112) that Aristotle might think that being in distinct positions at distinct instants is an incidental property of continuous motion. This seems plausible, especially in light of the fact that Aristotle thinks that a point can only be in motion incidentally.

[29] Discursive thought or φάσις/ἀπόφασις/κατάφασις, characteristically involves accidental predication (see *DA* 430b26ff.). Apparently Aristotle's idea is that when we assert "The *x* is F" where "is F" is an accidental predication, two forms succeed each other in our souls. (Aristotle thinks that essential predications do not involve the concatenation of distinct forms.)

[30] I am assuming, here, that Aristotle is not abandoning his doctrine that an actually infinite quantity is untraversable in favor of the view that an actually infinite quantity is traversable *incidentally*, mainly because the latter doctrine would imply the existence of actual infinities, and because Aristotle's denial of actual infinities is amply attested. At the end of this paragraph, I give the sense in which an incidental traversal of a sequence is a traversal of a potential sequence.

[31] This is not to claim that Zeno's paradox relies of the segmentation not being optional, as I explain below.

[32] A step-change in velocity is one where there is an instantaneous shift between two velocities without passing through any intermediate velocities. Of course, the shift would not *itself* represent a change to Aristotle, since he does not allow

change at an instant. Rather, the step-change is understood in terms of periods of time that span this instant.

[33] Cf. *Phys*. 247b13

[34] At any rate, Aristotle is problematizing something that should be quite simple. If acceleration from a stand is conceived of as simply the traversal of a continuum of instantaneous velocities starting from zero, then it should be no more problematic than the traversal of a spatial continuum from any given point of origin. Of course, Aristotle denies the existence of instantaneous velocities, so this simple approach is closed to him.

[35] At *Physics* 7.5 250a15-19, Aristotle adds the caveat that, due to friction, presumably, there will be some threshold force below which motion will not result. The example given is ship-hauling, where if it takes 100 men to move a ship 100 feet, it does not follow that one man can move a ship one foot.

[36] The sequence $d_n = 7/8^n$ (n = 1, 2, 3, …) was selected because it is consistent with a simple non-linear (parabolic) velocity function [i.e., $v(t) = 3(t - 1)^2$], and a simple distance/time function that passes through the points (0,0) and (1,1) [i.e., $d(t) = (t - 1)^3 + 1$]. The acceleration function is $a(t) = 6(t - 1)$.

[37] Sorabji, *Time, Creation And The Continuum*, 410-3.

[38] Niko Strobach, *The Moment of Change, A Systematic History in the Philosophy of Space and Time* (Dortrecht: Kluwer Academic Publishers, 1998), 52.

[39] Sorabji, *Time, Creation And The Continuum*, 411.

Bibliography

Annas, Julia. "Aristotle, Number and Time." *Philosophical Quarterly* 25 (April 1975): 97-113.

Archimedes. *The Works of Archimedes*. Translated by T. L. Heath. Cambridge: Cambridge University Press, 1897.

Barnes, Jonathan. *The Presocratic Philosophers*. London: Routledge & Kegan Paul, 1979.

Charlton, William. "Aristotle's Potential Infinites." In *Aristotle's Physics: A Collection of Essays*, ed. Lindsay Judson, 129-49. Oxford: Oxford Clarendon Press, 1995.

Corish, Denis. "Aristotle's Attempted Derivation of Temporal Order From That of Movement and Space." *Phronesis* 21 (1976): 241-251.

Frede, Michael. "Individuals in Aristotle." In *Essays in Ancient Philosophy*, 49-71. Minneapolis: University of Minnesota Press, 1987.

_____. "Substance in Aristotle's Metaphysics." In *Essays in Ancient Philosophy*, 72-80. Minneapolis: University of Minnesota Press, 1987.

Haslanger, Sally. "Persistence Through Time." In *The Oxford Handbook of Metaphysics*, eds. Michael J. Loux and Dean W. Zimmerman, 315-354. Oxford: Oxford University Press, 2003.

Hintikka, Jaakko. "Aristotelian Infinity." *Philosophical Review* 75 (April 1966): 197-218.

Hussey, Edward. *Aristotle's Physics III & IV*. New York: Oxford Clarendon Press, 1983.

Knorr, Wilbur R. "Infinity And Continuity." In *Infinity And Continuity In Ancient And Medieval Thought*, ed. Norman Kretzmann, 112-145. Ithaca: Cornell University Press, 1982.

Kosman, L. A.. “Aristotle’s Definition of Motion,” *Phronesis* 14 (1969): 40-62.

Kripke, Saul. *Naming and Necessity*. Cambridge, MA: Harvard University Press, 1972.

Lear, Jonathan. “Aristotelian Infinity,” *Proceedings of the Aristotelian Society* 80 (1979): 187-210.

Leibniz, Gottfried Wilhelm. *Leibniz: Selections*. Edited by P. P. Wiener. New York: Scribner, 1951.

Long, A. A. and Sedley, D.N.. *The Hellenistic Philosophers, Volume 1, Translations of the principal sources with philosophical commentary*. Cambridge: Cambridge University Press, 1987.

Miller, Fred D. "Aristotle on the Reality of Time," *Archiv für Geschichte der Philosophie* 56 (1974): 132-55.

Mueller, Ian. “Aristotle on Geometrical Objects,” *Archiv für Geschichte der Philosophie* 52 (1970): 156-71.

Owen, G E L.. “Aristotle on Time,” in *Motion and Time, Space and Matter*, eds. P. Machamer and R. Turnbull, 3-27. Columbus: Ohio State University Press, 1976.

Parfit, Derek. *Reasons and Persons*. Oxford: Oxford Clarendon Press, 1984.

Plantinga, Alvin. “Actualism and Possible Worlds.” *Theoria* 42 (1976): 139-60.

Reesor, M.E.. “The Stoic Concept of Quality,” *American Journal of Philology* 75 (1954): 40-58.

Rist, J.M.. “Categories and their Uses” in *Stoic Philosophy*, ed. J.M. Rist (Cambridge: Cambridge University Press, 1969), 152-72.

Russell, Bertrand. *Our Knowledge of the External World*. New York: W. W. Norton & Company, Inc., 1929.

Sedley, David. “The Stoic Criterion of Identity,” *Phronesis* 27 (1982): 255-75.

Sklar, Lawrence. “Up and Down, Left and Right, Past and Future,” *Nous* 15 (1981): 111-29.

Sorabji, Richard. *The Philosophy of the Commentators: 200-600 AD, A Sourcebook, Volume 3, Logic and Metaphysics*. London: Duckworth, 2005.

_____. *Time, Creation And The Continuum: Theories In Antiquity And The Early Middle Ages*. Ithaca: Cornell University Press, 1983.

Strobach, Niko. *The Moment of Change, A Systematic History in the Philosophy of Space and Time*. Dortrecht: Kluwer Academic Publishers, 1998.

Vlastos, Gregory. "Zeno's Race Course," *Journal of the History of Philosophy* 4 (1966): 95-108.

Waterlow, Sarah. "Aristotle's Now," *Philosophical Quarterly* 34 (April 1984): 104-28.

_____. "Instants of Motion in Aristotle's *Physics* VI," *Archiv für Geschichte der Philosophie* 65 (1983): 128-46.

_____. *Nature, Change and Agency in Aristotle's Physics*. Oxford: Oxford Clarendon Press, 1982.

White, Michael J. *The Continuous and the Discrete*. New York: Oxford Clarendon Press, 1992.

Zimmerman, Dean. "Temporary Intrinsics and Presentism," in *Metaphysics: The Big Questions*, eds. D. Zimmerman and P. Van Inwagen, 206-219. Oxford: Basil and Blackwell, 1998.

Vita

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